CSCI 260 Test Preparation

Midterm 1

Big O Rules

- Bring to the test your printed Cheat Sheet (includes the Big-O Rules, definition of Big-O, and the Master Theorem). You can also write your own notes on the same sheet.

  If you don't bring your own sheet, I will bring spares.

Big-O Rules:

1. Transitivity of Big O

"and" "implies"

$f(n) \in O(g(n)) \wedge g(n) \in O(h(n)) \rightarrow f(n) \in O(h(n))$

2. Log Domination (powers of n always dominate powers of logs of n)

$\log^r n \in O(n^s)$ $\forall$ constants r, s where $s > 0$

3. Polynomial Rule

$p(n) \in O(q(n))$ whenever $p(n)$ and $q(n)$ are polynomials, and the degree of $q(n)$ is $\geq$ degree of $p(n)$.

[note: the first coefficient of $q(n)$ must be positive]

4. Product Rule

① $f_1(n) \in O(g_1(n))$

② $f_2(n) \in O(g_2(n))$

$\wedge \rightarrow f_1(n) \cdot f_2(n) \in O(g_1(n) \cdot g_2(n))$

by ①, ② Product Rule

5. Constant Factors Rule

$c \cdot f(n) \in O(f(n))$ $\forall$ constant $c > 0$

6. Log Base is Irrelevant

$\log_a(n) \in O(\log_b n)$ $\forall a, b > 0, a \neq 1, b \neq 1$

7. Reciprocal Rule

$f(n) \in O(g(n)) \rightarrow \frac{1}{g(n)} \in O\left(\frac{1}{f(n)}\right)$

8. Sum Rule

$f_1(n) \in O(g(n))$ ∧ $f_2(n) \in O(g(n))$ → $f_1(n) + f_2(n) \in O(g(n))$

9. Negative Terms Rule

$f(n) \in O(g(n))$ ∧ $h(n) > 0$ → $f(n) - h(n) \in O(g(n))$

10. Side Proof

Any Big-O claim proved elsewhere on the page can be introduced into the proof.

NEW:

11. Order Preserving function applied

$h(n)$ an order-preserving function ∧ $f(n) \in O(h(g(n)))$

→ $h(f(n)) \in O(h(g(n)))$

[h is order-preserving if $\forall x, y > 10$

$x \leq y \rightarrow h(x) \leq h(y)$]

Note: lg is an order-preserving function

Examples

Q1. Prove using the rules of Big O that

$$4n + 3 \lg n - 17 \in O(n)$$

Answer.

Claim: $4n + 3 \lg n - 17 \in O(n)$

Proof: ① $4n \in O(n)$ Constant Factors

② $\lg n \in O(n)$ Log domination

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②	$\lg n \in O(n)$	Log domination
③	$3\lg n \in O(\lg n)$	Constant Factors
④	$3\lg n \in O(n)$	③, ② Trans

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②	$\lg n \in O(n)$	Log domination
③	$3\lg n \in O(\lg n)$	Constant Factors
④	$3\lg n \in O(n)$	③, ② Trans
⑤	$4n + 3\lg n \in O(n)$	①, ④ Sum Rule
⑥	$4n + 3\lg n - 17 \in O(n)$	⑤, Negative Terms



Q2: Prove using the rules of Big O that (7)

$$\lg \lg n \in O(n)$$

Answer. Claim: $\lg \lg n \in O(n)$

Proof: ①

Q2: Prove using the rules of Big O that (8)

$$\lg \lg n \in O(n)$$

Answer. Claim: $\lg \lg n \in O(n)$

Proof: ① $\lg n \in O(n)$ Log Dom

② $\lg \lg n \in O(\lg n)$ ①, lg is order-preserving

Q2: Prove using the rules of Big O that (9)

$$\lg \lg n \in O(n)$$

Answer. Claim: $\lg \lg n \in O(n)$

Proof: ① $\lg n \in O(n)$ Log Dom

② $\lg \lg n \in O(\lg n)$ ①, lg is order-preserving

③ $\lg \lg n \in O(n)$ ②, ① Transitivity. ▨