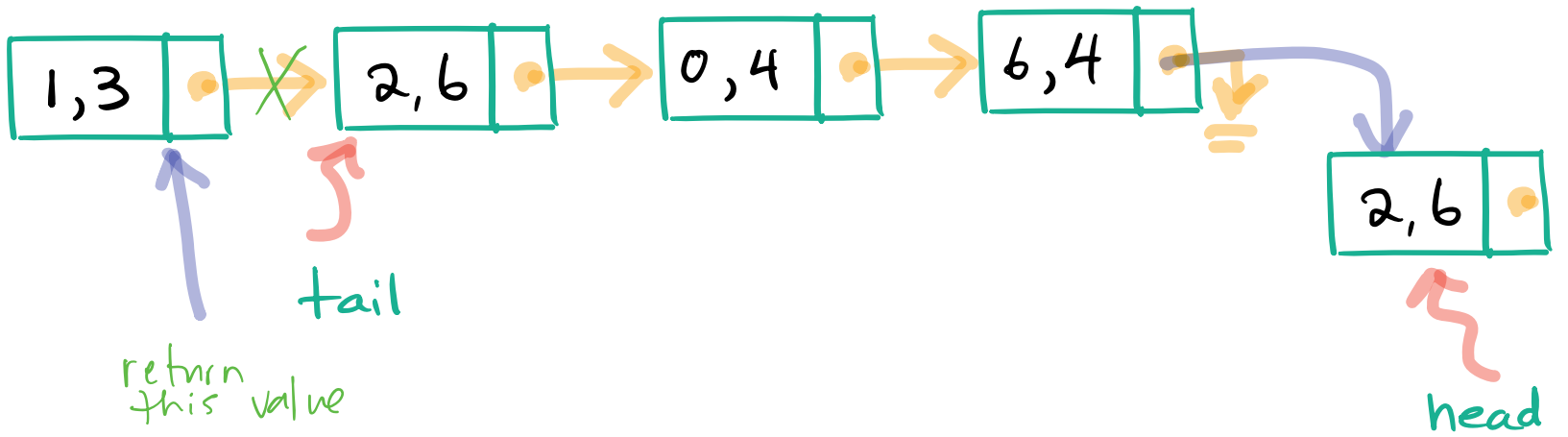
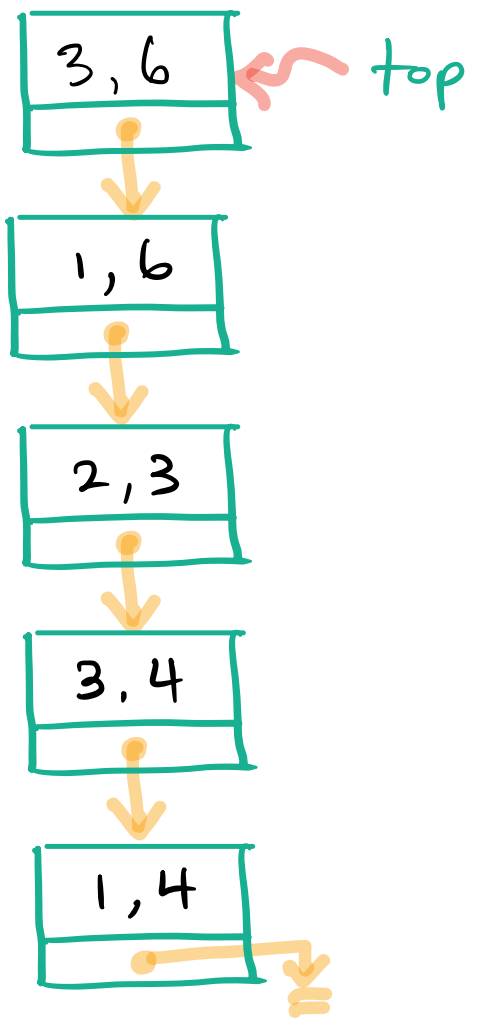
Review: Queues and Stacks

Recall that we sometimes want to store elements of data in a DS that accesses those elements in...

FIFO order = "first-in - first-out"



LIFO order = "last-in - first-out"



Also recall what a "cross product" of 2 sets is:

$$\{a,b\} \times \{1,2,3\} = \{(a,1),(a,2),(a,3),$$
$$(b,1),(b,2),(b,3)\}$$

Note that a cross-product is a set of ordered pairs. In Graph Theory, we abuse notation somewhat and often use $(1,4)$ to denote an unordered pair

— it will be clear from the context which one we mean:

Directed Graphs = ordered pair

Undirected Graphs = unordered pair

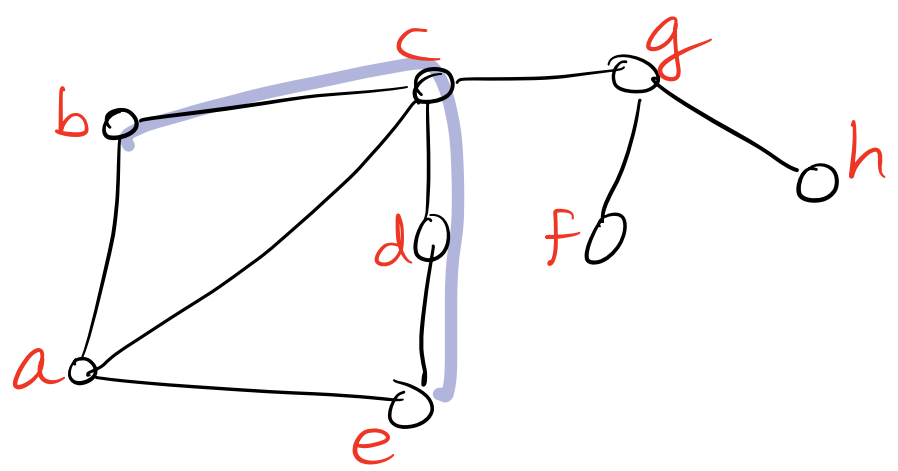
we will start with these

Graph = Undirected Graph (for now)

Graphs

A graph is a set of vertices V and an edge set $E \subseteq V \times V$.

(Undirected) graph $G = (V, E)$



By convention,

$|V| = n$

$|E| = m$

$V = \{a, b, c, d, e, f, g, h\}$

$E = \{(a,b), (a,e), (b,c), (d,c), (e,d), (c,g), (f,g), (g,h), (a,c)\}$

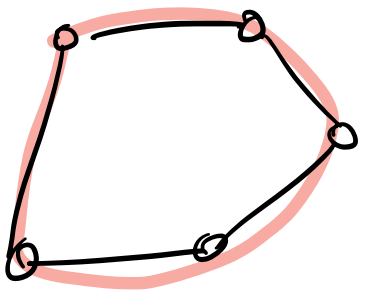
Def^n path is a sequence $v_0, v_1, \cdots, v_t$, where

1. $v_i \in V \quad \forall \; 0 \le i \le t$; and
2. $(v_i, v_{i+1}) \in E \quad \forall i, \; 0 \le i < n$

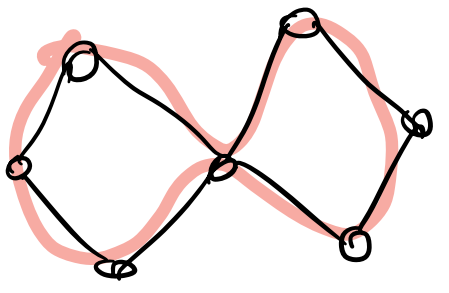
The path is simple if $v_i \neq v_j$ whenever $i \neq j$

That path is also a $v_0 - v_t$ path.

A path where $v_0 = v_t$ is a cycle, and is a simple cycle if it is simple except at endpoints.



Simple cycle



a cycle that is not simple is sometimes called a circuit.

Note: if $G = (V_G, E_G)$

then $V(G) = V_G$

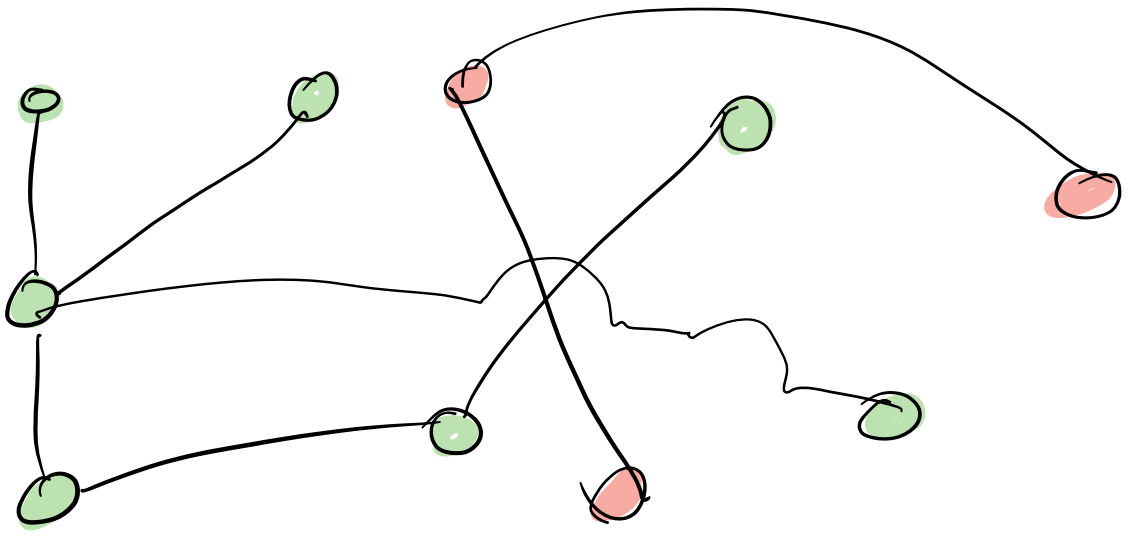
and $E(G) = E_G$.

Defn: The length of a path is the number of edges on the path.

Defⁿ The distance between vertices u, v in a graph G is the smallest length of any u-v path in G.

(If the edges have weights, then the distance is the least sum of any weighted u-v path)

The shortest u-v path (shortest path from u to v) is the path with the smallest length.



Defn: A graph is connected if $\forall u, v \in V$

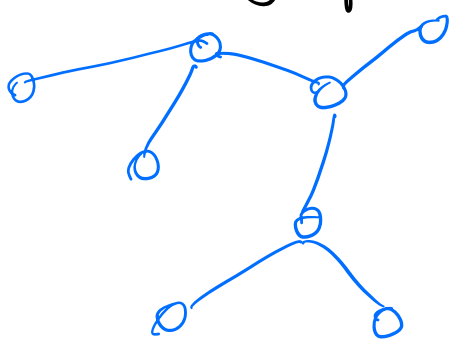
$\exists$ a u-v path in G.

Defn: A connected component of a graph G is a maximal subgraph of G that is connected.

Defn: A subgraph of $G = (V, E)$ is a graph $G' = (V', E')$ such that $V' \subseteq V$ and $E' \subseteq E$. (also, $E' \subseteq V' \times V'$)

Defn: An induced subgraph $G'' = (V'', E'')$ of $G = (V, E)$ has $V'' \subseteq V$, and $E'' = E \cap (V'' \times V'')$

Defn A tree is an acyclic connected graph.



T_1

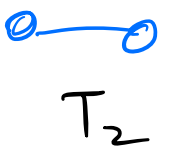
How many edges are in a tree?

$|V| = n$

$|E| = m$

$m = n - 1$

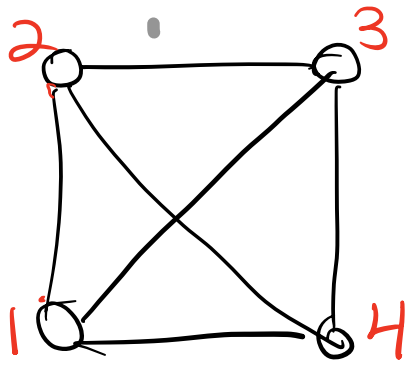
Graph Representation.



T_2

T_0

T_{-1}



	1	2	3	4
1	0	1	0	0
2	1	0	1	1
3	0	1	0	1
4	0	1	1	0

Adjacency Matrix representation.

Space = $\Theta(n^2)$.