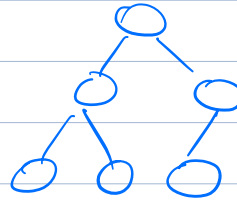


Mergeable Heaps

25-11-4

Binomial Heaps

Recall Binary Heap



- rigid structure

- heap ordered

We pay for the rigidity of the structure somewhat
How?

Binomial Heap

- heap ordered collection of trees
- no 2 trees have same degree
- if insertion/deletion of a node leads to 2 trees of same degree d
join them to get a tree of degree $d+1$

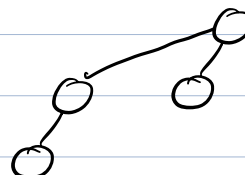
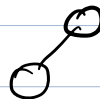
0...1 nodes



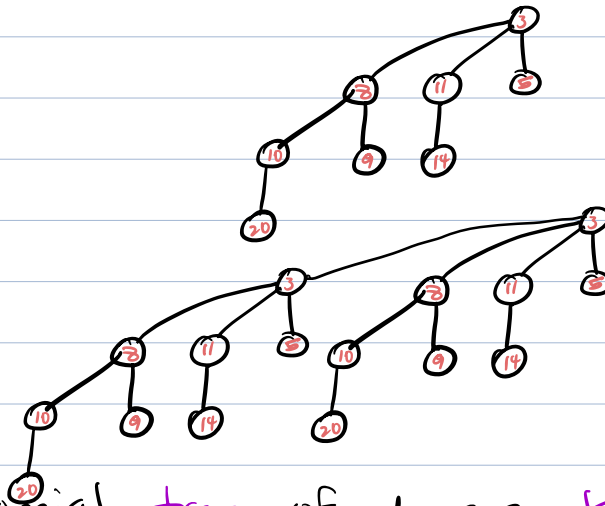
2...3 nodes



4...7 nodes

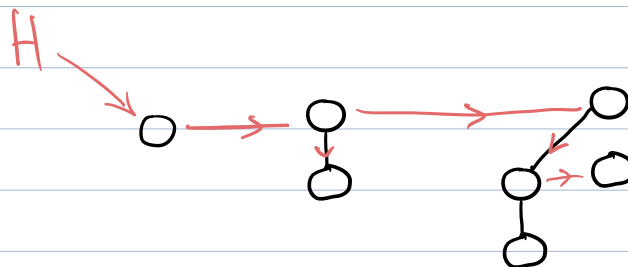
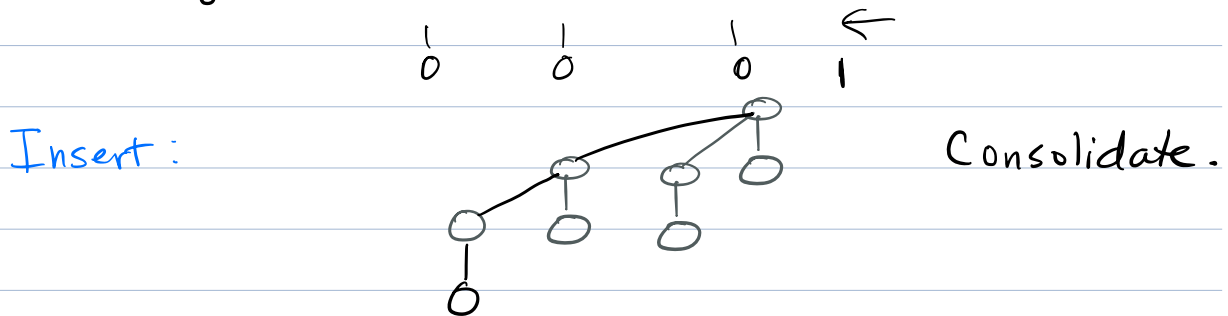


8 ... 15 nodes



Theorem: A binomial tree of degree k
 has 2^k nodes
 k height
 $\binom{k}{i}$ nodes at depth i

Proof: By induction.



Delete Min: d x tree merges, where each tree merge op is $\Theta(1)$.

$d \leq \lg n$ $\therefore$ takes $\Theta(\lg n)$ time.

DecreaseKey: $\Theta(\text{height of tree Key is in})$

$h \leq d \leq \lg n$ by Thm.

$\therefore \in \Theta(\lg n)$