Assignment 2
WITH SOLUTIONS

Oct 15, 2025
Due Oct 23

1. [10] Give a Dynamic Programming recursive formula for "Maximum Bitonic Subsequence".
   I.E. find the cardinality of the largest not-necessarily-contiguous subsequence of a given input sequence $A = [a_1, a_2, \ldots, a_n]$ of integers.
   [a sequence is bitonic if its values rise and then fall. 8 1 4 5 3 9 2 7 6 9 2 -the red numbers are a bitonic subsequence. Either the rising or the falling can be empty.]

   Ensure the recursive formula does not have cyclic dependencies by identifying an evaluation order

Solution: First, for each element a_i calc and memoize
- longest increasing subseq that ends at a_i
- longest decreasing subseq that starts at a_i
- Note: This solution assumes STRICTLY increasing, then STRICTLY decreasing. The alternative is also acceptable, as the ambiguity is in the question.

$$Inc[i] = \begin{cases} 1 \text{ if } i = 1 \\ \max_{\substack{j<i \\ \text{where} \\ a_j < a_i}} \left(Inc[j] \right) + 1, \text{ otherwise} \end{cases}$$

$$Dec[i] = \begin{cases} 1 & \text{if } i = n \\ \max_{\substack{j > i \\ \text{where} \\ a_j > a_i}} (Dec[j]) + 1, & \text{otherwise} \end{cases}$$

$$Bit[i] = Inc[i] + Dec[i] - 1$$

You can evaluate in the order

" for each $i = 1, 2, \ldots n$

1. evaluate $Inc[i]$ and $Dec[n+1-i]$

for each $i = 1, 2, \ldots n$

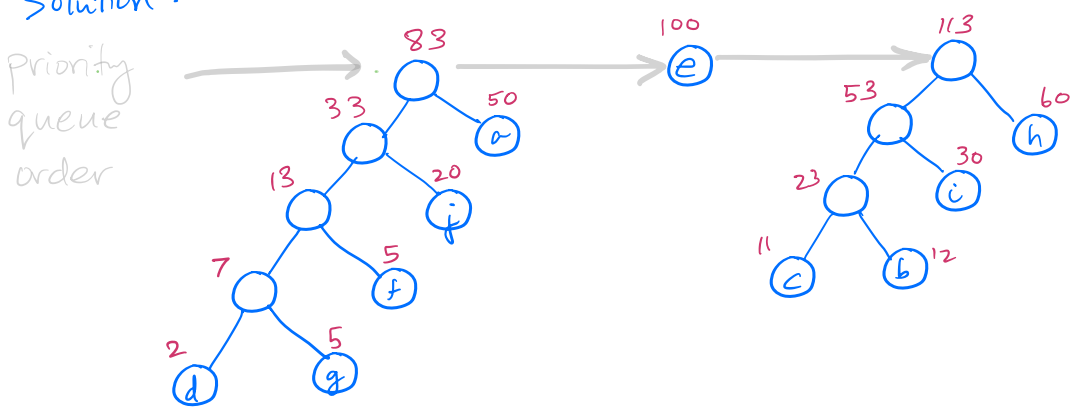
1. evaluate $Bit[i]$ "

2. [1 mark] Let the letters and frequencies be as given below by A and F, respectively. What are the contents of the Priority Queue when only three trees remain?

A =	a	b	c	d	e	f	g	h	i	j
F =	50	12	11	2	100	5	5	60	30	20

Give the three trees, with key values on each node of each tree (ie priority-queue key values).

Solution:


priority queue order
83
33
50
a
13
20
j
7
5
f
2
5
d
g
100
e
113
53
60
h
23
30
i
11
12
c
b