CSCI 429 Assignment 1 Due Sept 9, 2025

1. [5] ADT Array is given below:

```
init()
set(int i, int val)
    // precondition: val ≠ NO_VAL,
    // where NO_VAL is a reserved value
int get(int i)
    // returns NO_VAL if there has
    // been no set(i, val) since
    // last init(); returns
    // val in last set(i, val)
    // operation otherwise
```

Give pseudocode for an implementation of Array that runs in $O(1)$ for each operation

2. [5] Let $x_1 < x_2 < \cdots < x_n$ be an ordered set of elements (or keys) and let

$f[i]$ = number of times x_i is searched for in the lifetime of the BST that stores the elements.

The cost "Cost[T]" of a Binary Search Tree T that stores the elements is the total number of element-comparisons executed over all searches.

OptCost[1 .. n] is the optimal (lowest) cost of any BST that stores the elements $\{x_1, \ldots, x_n\}$ and, more generally,
OptCost[i, k] is the optimal cost of any BST on the elements $\{x_i, \ldots, x_k\}$, for any $1 \le i \le k \le n$, under the given frequency assumptions (i.e. f).

Prove that

$$\mathrm{OptCost}[i,k] = \sum_{i \le j \le k} f[j] + \min_{i \le r \le k} \left\{ \begin{matrix} \mathrm{OptCost}[i, r-1] \\ + \\ \mathrm{OptCost}[r+1, k] \end{matrix} \right\}$$

Hint: Use induction.