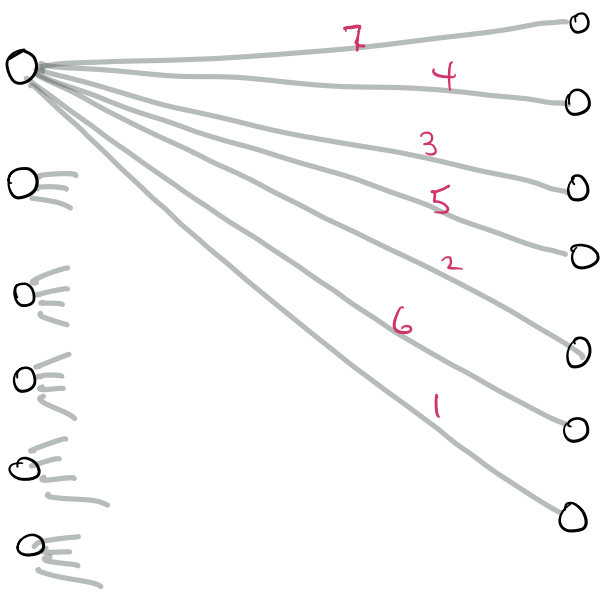
Greedy Alg: Gale Shapley 1108

Doctors Internships 4?



Each Doctor ranks the internships

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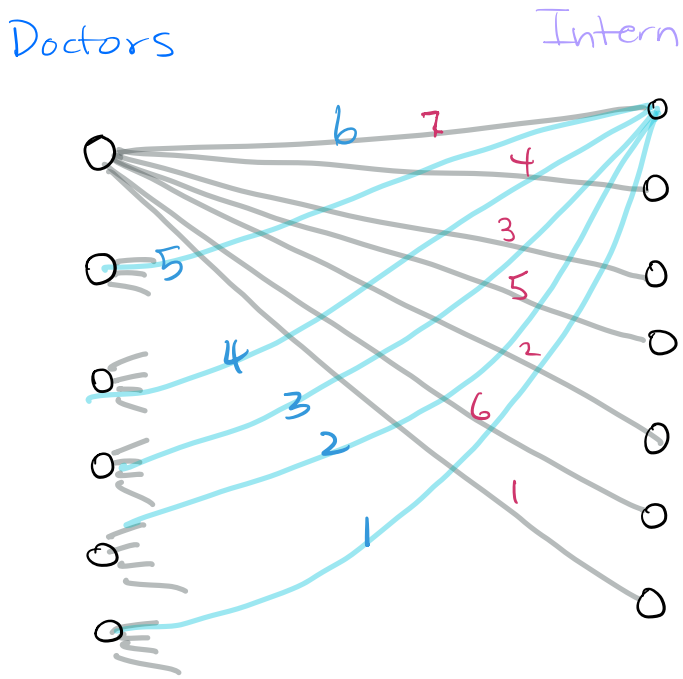
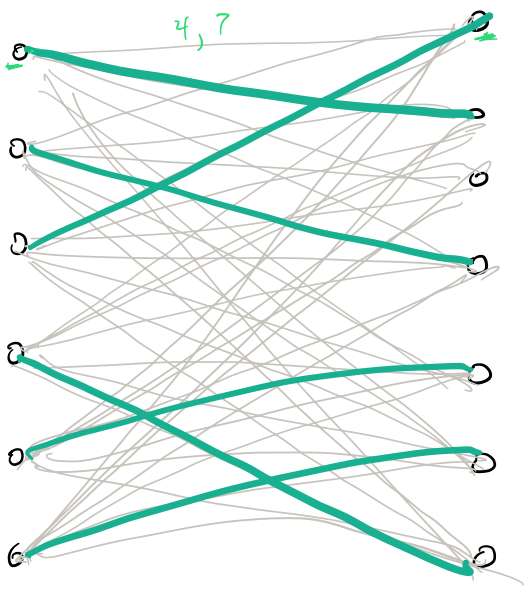


Each Doctor ranks the internships

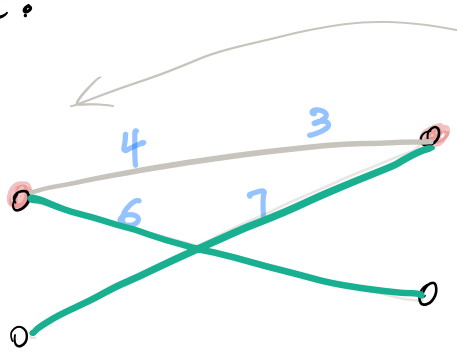
Each Hospital (Internship) ranks the doctors



Find a "stable" matching

- no doctor prefers a hospital that also prefers that doctor

unstable:



unstable pair — prefer each other to whomever they are matched to.

Does a stable matching always exist?

[History: This problem actually emerged out of the massive doctor-hospital matching problem every year in the US, "National Resident Matching Program", 1950's.

1952 — Boston Pool algorithm was used

1962 — Gale & Shapley formalized the problem, the alg., and proved it correct.

Shapley: 2012 Nobel Prize in economics (w/ Roth)

It is used for: kidney-donor
faculty-university (France)
university-student (Germany)
ship-sailor (US Navy)
...etc]

Yes, a stable solution always exists!

Greed must be carefully applied, as just "resolve some instability" iteratively can lead to infinite loops...

Doctors	q	r	s	t
top choice:	A	A	B	D
	B	D	A	B
	C	C	C	C
	D	B	D	A

Hospitals	A	B	C	D
top choice:	t	r	t	s
	s	t	r	r
	r	q	s	q
	q	s	q	t

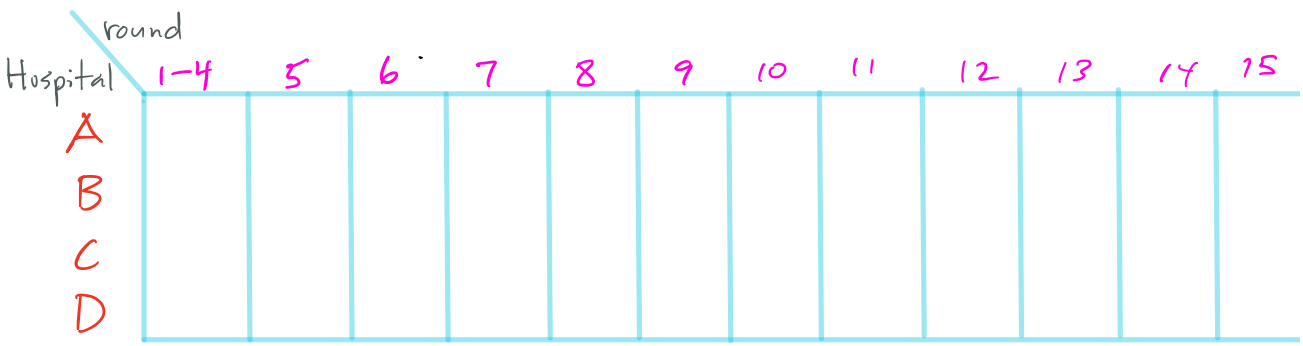
Round: - An arbitrary unmatched hospital X makes an offer to top-choice doctor y who has not rejected X

- If y is unmatched — y matched with X
- If y prefers X to y's match — y matched with X
- If y prefers its current match — y rejects X

Keep executing rounds until all hospitals are matched.

Doctors	q	r	s	t
top choice:	A	A	B	D
	B	D	C	B
	C	C	A	C
	D	B	D	A

Hospitals	A	B	C	D
top choice:	t	r	t	s
	s	t	r	r
	r	q	s	q
	q	s	q	t



Doctors	q	r	s	t
top choice:	A	A	B	D
	B	~~D~~	C	B
	C	~~C~~	~~A~~	~~C~~
	D	~~B~~	~~D~~	~~A~~

Hospitals	A	B	C	D
top choice:	~~t~~	~~r~~	~~t~~	~~s~~
	~~s~~	t	~~r~~	~~r~~
	r	q	s	q
	q	s	q	t

Hospital \ round	1-4	5	6	7	8	9	10	11	12	13	14	15
A	t		s	s	s		r	r				
B	r	r	r		t	t	t	t				
C	s	t	t	t		s	s	s				
D	q	q	q →	r	r	r		q				

Thm: The Gale-Shapley Algorithm results in a stable matching

Proof: It continues until there are no more instabilities, so we need only show that it terminates, which follows from the next theorem. ▨

Thm: The Gale-Shapley Alg terminates within n^2 rounds.

Proof:

hospital \ round	1-4	5	6	7	8	9	10	11	12	13	14	15
A	t		s	s	s		r	r				
B	r	r	r		t	t	t	t				
C	s	t	t	t		s	s	s				
D	q	q	q	r	r	r		q				

↑

∃ exactly one new appearance of a doctor in any round. (column)

∀ rows, at most n hospitals can appear.

⇒ at most n rounds assign a new doctor to A

▨