Greedy Alg: Gale Shapley   1108

Doctors   Internships

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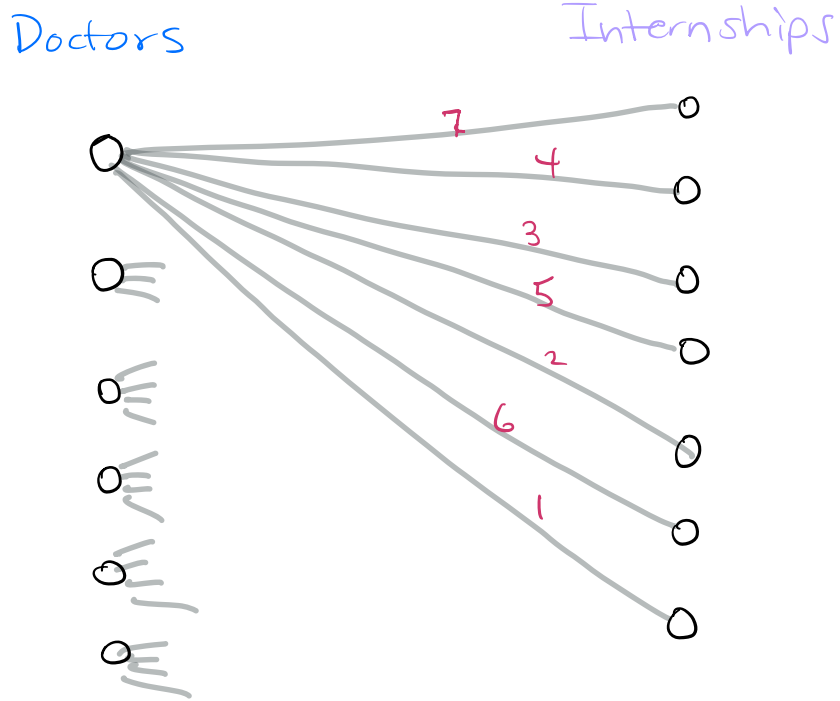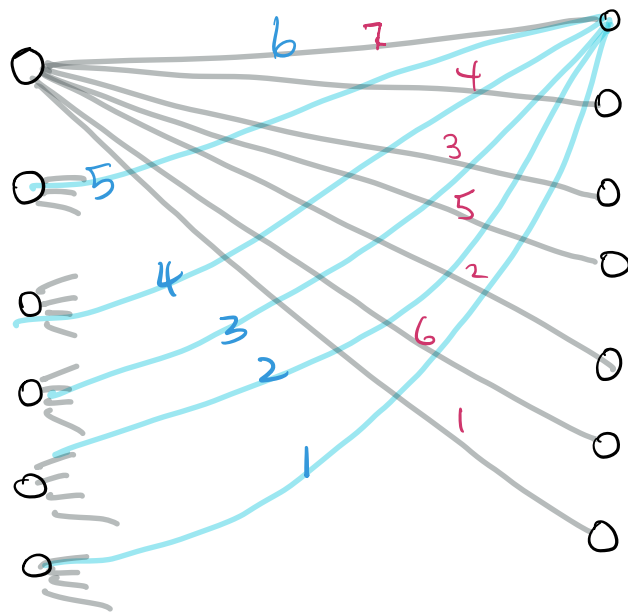3
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Each Doctor ranks the internships

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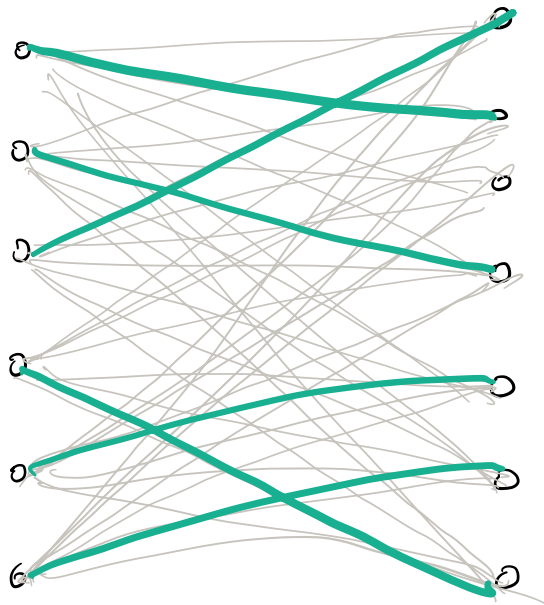
Doctors   Internships
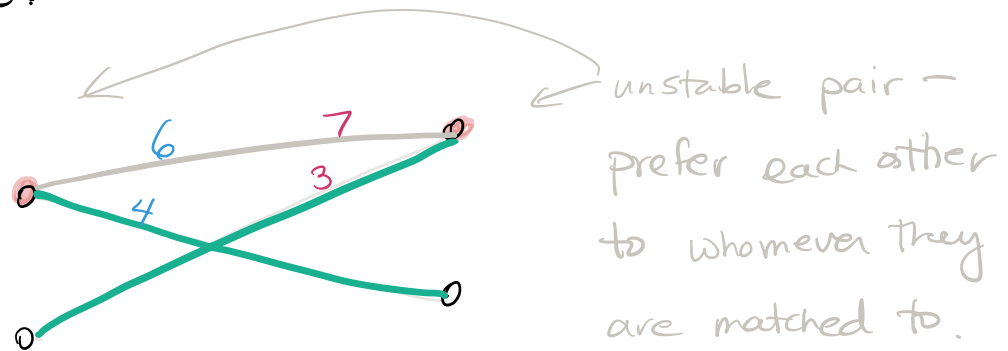

Each Doctor <u>ranks</u> the internships
Each Hospital (Internship) ranks the doctors

Find a "stable"
matching
— no doctor
prefers a hospital
that also prefers
that doctor

unstable:

unstable pair —
prefer each other
to whomever they
are matched to.

Does a stable matching always exist?

[History: This problem actually emerged out of the massive doctor-hospital matching problem every year in the US, "National Resident Matching Program", 1950's.

1952 — Boston Pool algorithm was used

1962 — Gale & Shapley formalized the problem, the alg., and proved it correct.

Shapley: 2012 Nobel Prize in economics (w/ Roth)

It is used for: Kidney-donor
faculty-university (France)
university-student (Germany)
ship-sailor (US Navy)

... etc
]

Yes, a stable solution always exists!
Greed must be carefully applied, as just "resolve some instability" iteratively can lead to infinite loops...

Doctors	q	r	s	t		Hospitals	A	B	C	D
top choice:	A	A	B	D		top choice:	t	r	t	s
	B	D	A	B			s	t	r	r
	C	C	C	C			r	q	s	q
	D	B	D	A			q	s	q	t

Round :— An arbitrary unmatched hospital X makes an offer to top-choice doctor y who has not rejected X

- If y is unmatched   y matched with X
  If y prefers X to y's match   y matched with X
  If y prefers its current match   y rejects X

Keep executing rounds until all hospitals are matched.

Doctors	q	r	s	t
top choice:	A	A	B	D
	B	D	A	B
	C	C	C	C
	D	B	D	A

Hospitals	A	B	C	D
top choice:	t	r	t	s
	s	t	r	r
	r	q	s	q
	q	s	q	t

A — t

B — r

C —