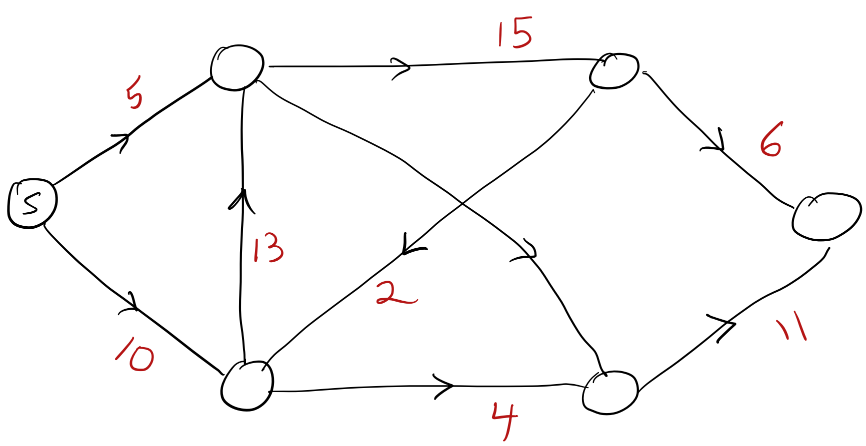
Network Flows

10.25

Given: A directed graph $G=(V,E)$
with edge capacities $C: E \to \mathbb{R}$
and distinct source vertex (no in-edges) s
and distinct sink vertex (no out-edges) t

Find: Max amount of flow from s to t

?



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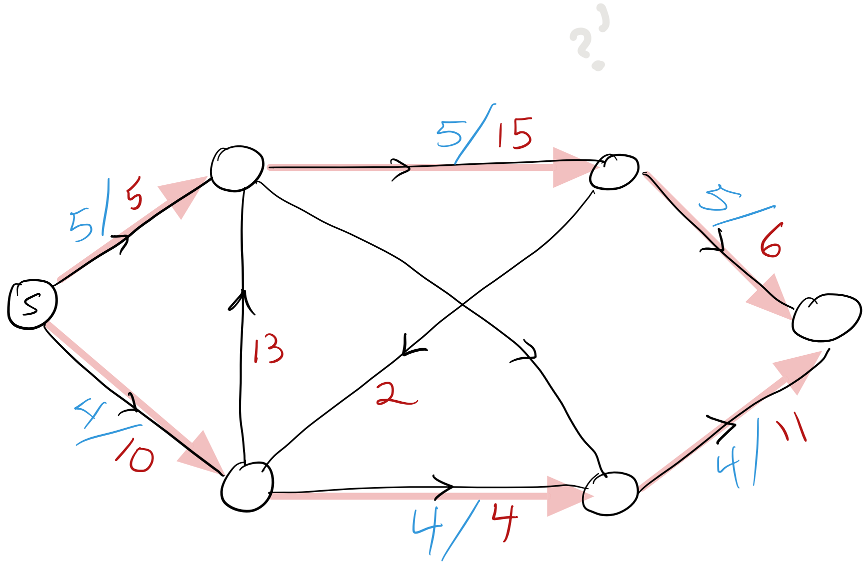

Total flow = flow out from s
= flow into t
= 9

Max?

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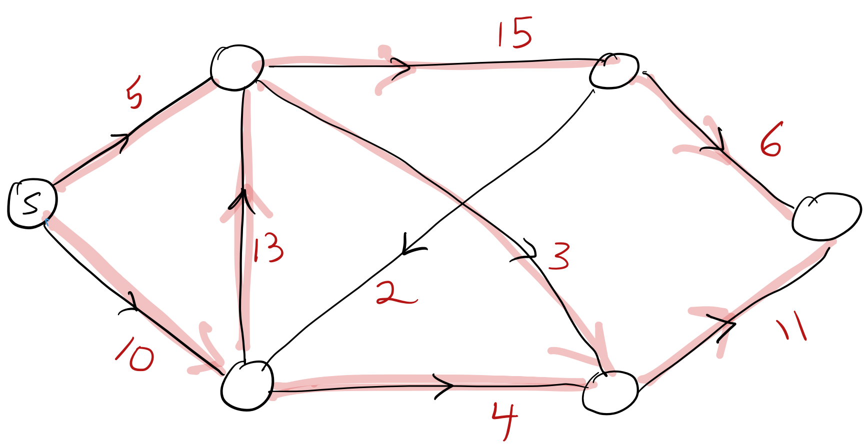
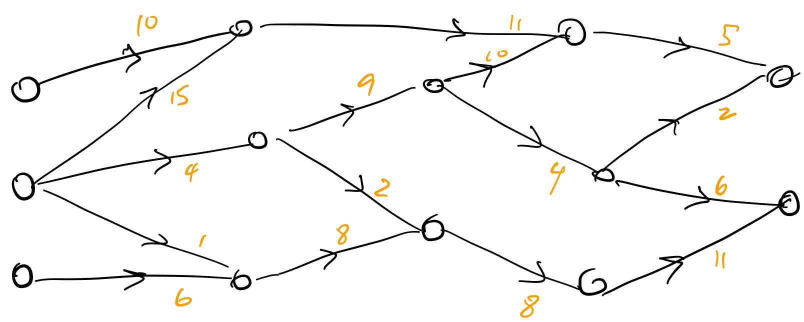


Multi-Source / Multi sink ?

Not a problem.



Defn Flow

A flow is an assignment of values $f: E \to \mathbb{R}$ such that:

$\forall (u,v) \in E, \quad f(u,v) \le c(u,v)$ Obey capacity constraints

$\forall v \in V, \; v \neq s \; v \neq t$

$$\sum_{u \in V} f(u,v) = \sum_{w \in V} f(v,w)$$

Conservation of Flow

Observe: max flow consists in maximizing

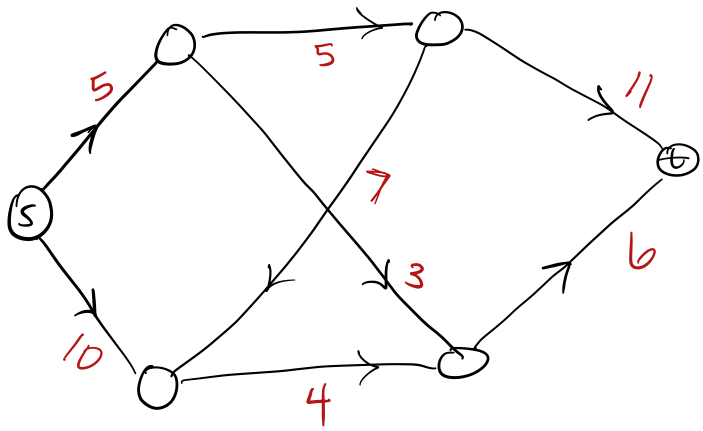
$$\sum_{u \in V} f(s,u)$$

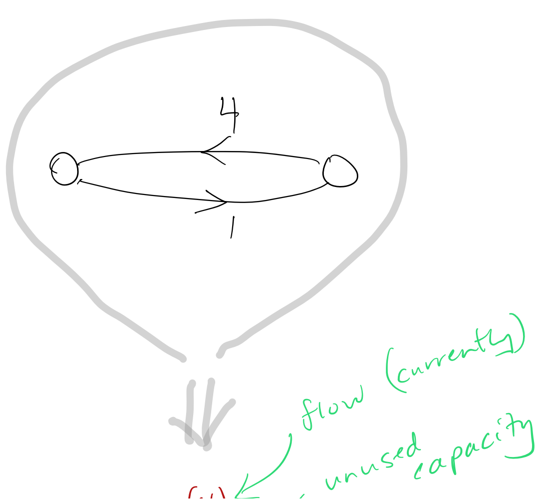
(Why not maximize $\sum_{w \in V} f(w,t)$?

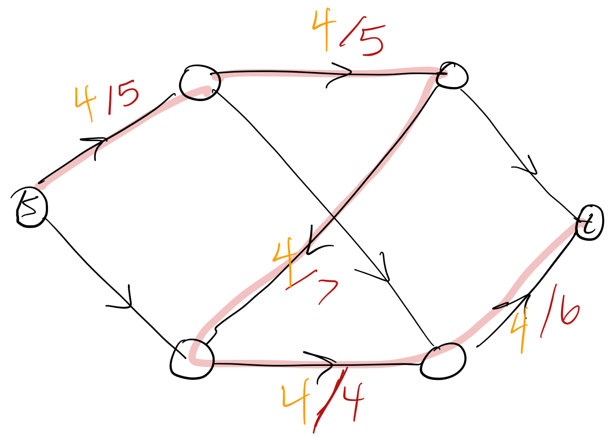
"Flow Augmenting Path" Alg : Ford & Fulkerson

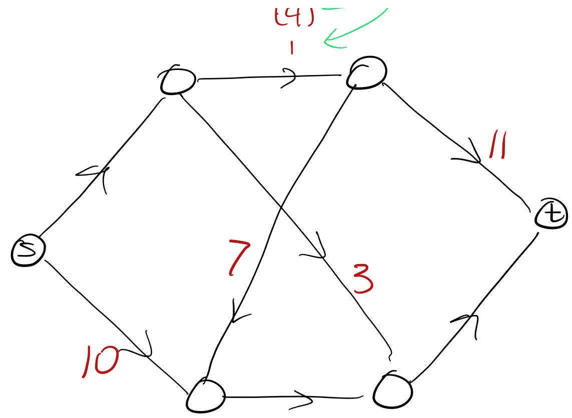
Iterative.

- Find a s-t path that has some unused capacity on all the forward edges, and positive flow on all the backwards edges
- Augment current flow with min value unused capacity (or positive flow on backwards edges) on all path edges.









Residual