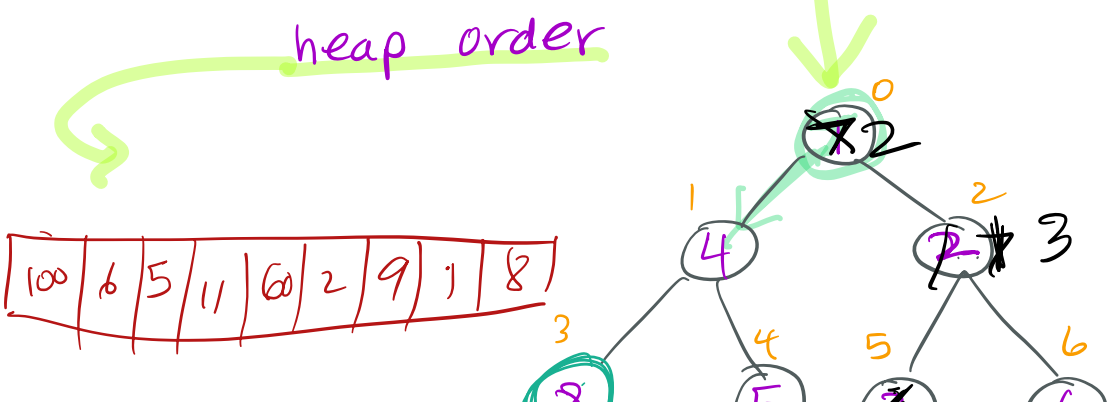
429 Binomial Heaps 10.4.

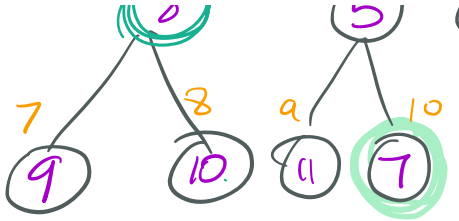
We are interested in Priority Queue ADT and will explore other options for their implementation.

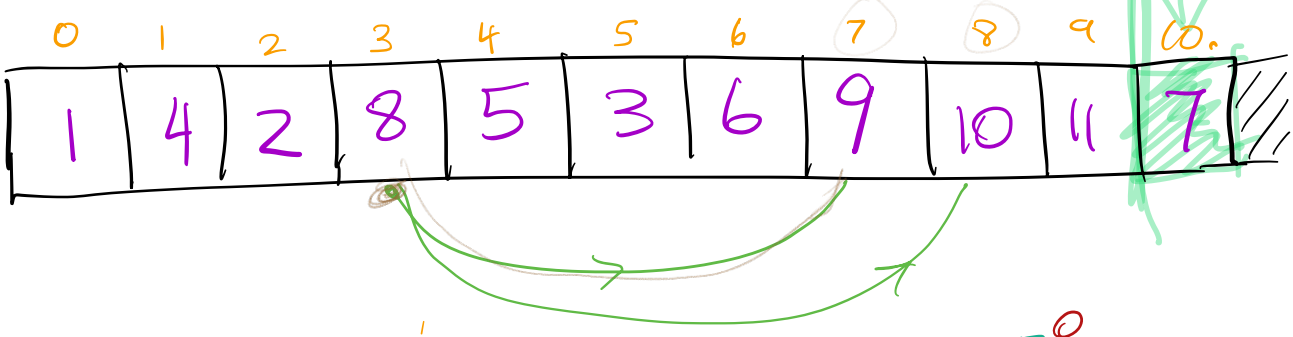
Procedure	Binary Heap: wc	Fibonacci Heap: Amortized	
MakeHeap	$\Theta(1)$	$\Theta(1)$	
Insert	$\Theta(\lg n)$	$\Theta(1)$	
Min	$\Theta(1)$	$\Theta(1)$	
ExtractMin	$\Theta(\lg n)$	$\Theta(\lg n)$	
Union	$\Theta(n)$	$\Theta(1)$	
DecreaseKey	$\Theta(\lg n)$	$\Theta(1)$	
Delete	$\Theta(\lg n)$	$\Theta(\lg n)$	

Recall: Binary Heap was studied in 260

– keep a complete binary tree in heap order

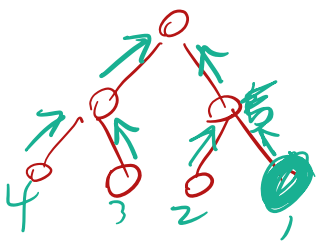






$left(i) = 2i + 1$

$right(i) = 2i + 2$



Binary Heaps are great, but they are not easily **Merged**

$\Theta(n)$

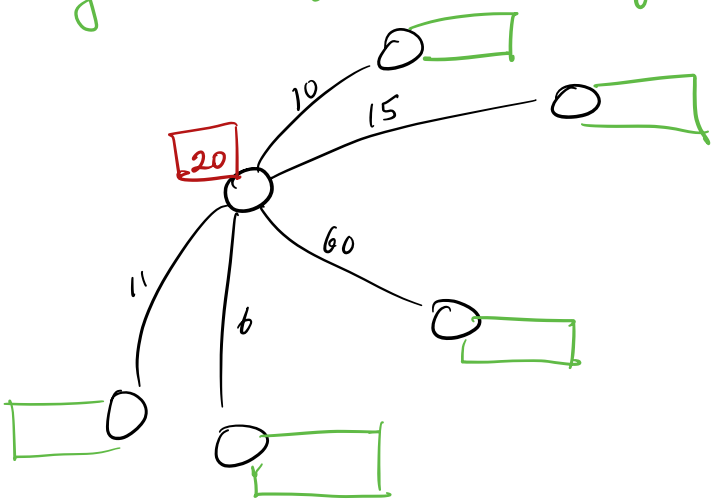
Many applications of Priority Queues require ability to take two PQs and merge them into one.

For the next few lectures, we will focus on this topic ... **Mergeable Heaps.**

Cormen, Leiserson, Rivest + Stein, 3rd Ed, Ch 19
(Fibonacci Heaps)

For applications such as in Graph Algorithms
(Single-Source Shortest Paths (Dijkstra's))
we may use many Decrease Key operations
- may far outnumber the ExtractMin ops.

Eg in Dijkstra's Alg



Dense graph
- may have
$\Theta(n)$ edges
to update
(Decrease Key)
for each
"permanent"
label found.

Fibonacci Heaps useful for
- Shortest Paths
- Min Spanning Tree

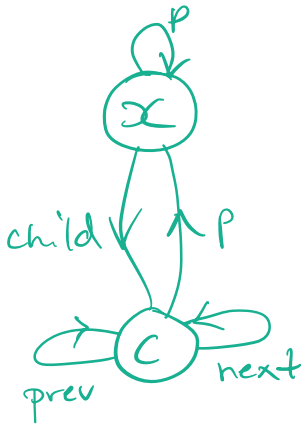
Would like simpler DS with same bounds, though.

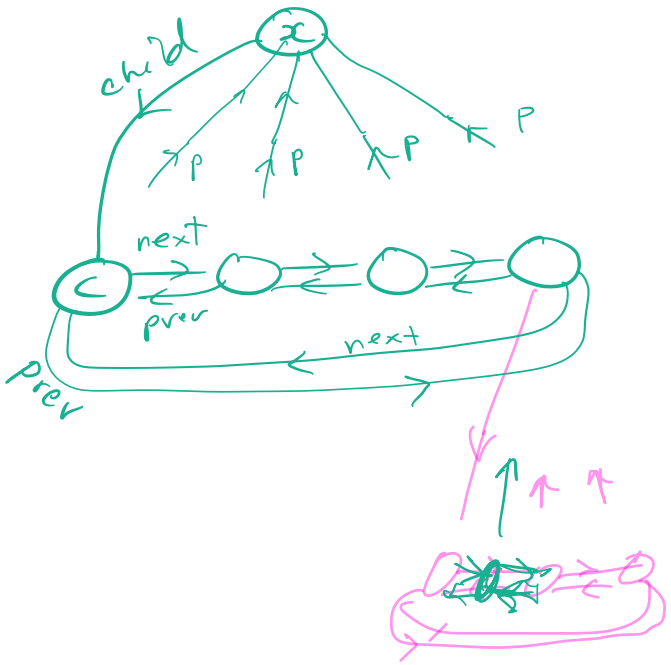
Fibonacci Heap

- collection of rooted trees
  that are min-heap ordered.
  (key at node $\leq$ keys in left- or right- subtree)

- each node has
  - pointer to parent $x \to p$
  - pointer to child $x \to child$.

  The children are in a doubly-linked circular list





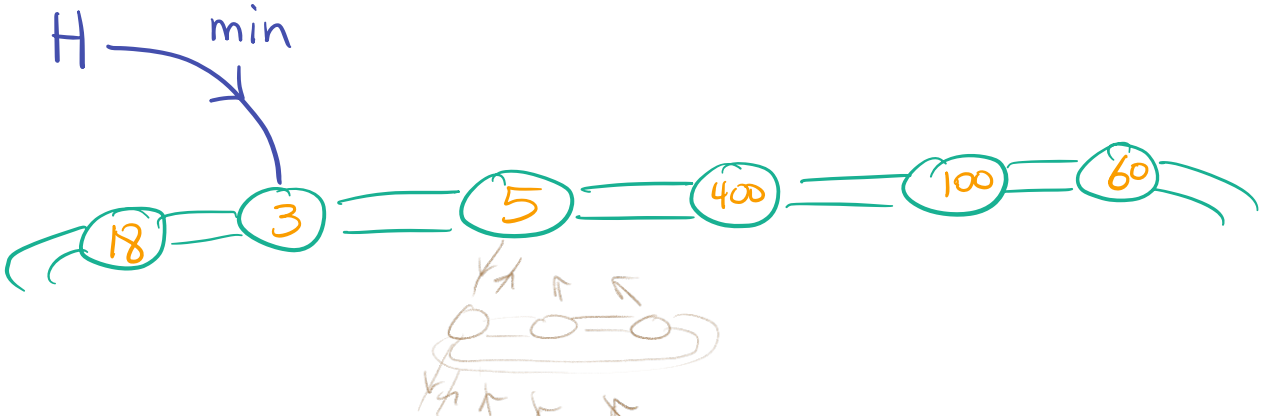
Circular doubly-linked child lists:
can be inserted into and combined in $O(1)$

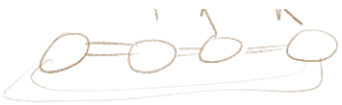
A Fibonacci Heap is a pointer to a root list

- the root it points to has min key value among all the roots in the list
- Furthermore, all the trees in the root list are trees formed by Fib-merges (below)
- Root list is like the child list of a node, but $x \to p == x$ $\forall$ roots x in the list.

$x \to$ degree = # children in x's child list

$x \to$ mark = "x has lost a child since it was made a child of another node"





for Fib Heap H: $H \to n$ = # nodes in H.

$t(H)$ = # roots in H's root list

$m(H)$ = # marked nodes in H

We will use the Potential Function method for amortized analysis

$$\phi(H) = t(H) + 2m(H)$$

$\Delta\phi$ = net potential changes to collection of heaps.

- a unit of potential can pay for a constant amount of work.

Max degree in our n-node Fib Heap will be denoted $D(n)$

When only the mergeable heap ops are supported, (no DecreaseKey or Delete)

$D(n) \leq \lfloor \lg n \rfloor$

When DecreaseKey and Delete are also used,

$D(n) \in O(\lg n)$

Fib Heap Ops

FibHeap.MaKe

$H \to n = 0$

$H \to min = NULL$

$\Phi(H) = 0.$

Cost $= \Theta(1)$

AmCost $= \Theta(1)$.

FibHeap.Insert(x)

// x→Key is already filled in

$x \to degree = 0$

$x \to p = x$

$x \to child = NULL$

Cost $= \Theta(1)$

$\Delta\Phi = 1$

$= \Theta(1)$.

if $H \to min == NULL$

create a root list that just contains x

$H \to min = x$

else

insert x into H's root list

if $x \to Key < H \to min \to Key$ $H \to min = x$.

$H \to n = H \to n + 1$

FibHeap.Min

return H→min

Cost =

ΔΦ =

FibHeap.Union (H_1, H_2)

// H_1 and H_2 will not be usable in future

// H will be their union

H = FibHeap.Make()

H→min = H_1→min

Concatenate root list of H_2 with root list of H.

if (H→min == NULL) or

(H_2→min ≠ NULL and H_2→min < H→min)

H→min = H_2→min

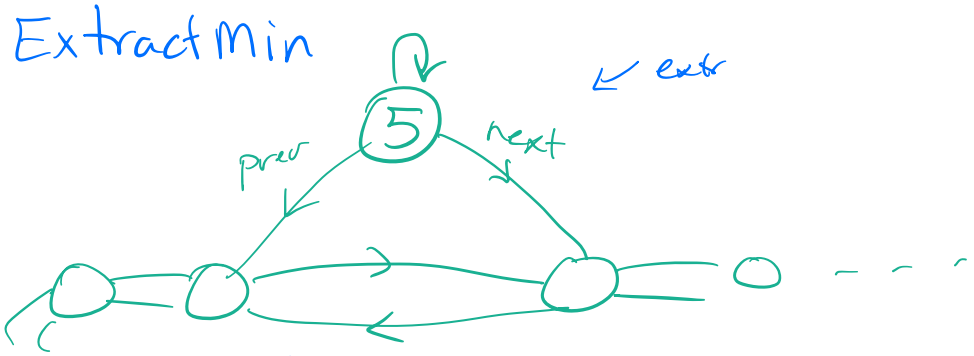
H→n = H_1→n + H_2→n

return H.

Cost =

ΔΦ =

ExtractMin

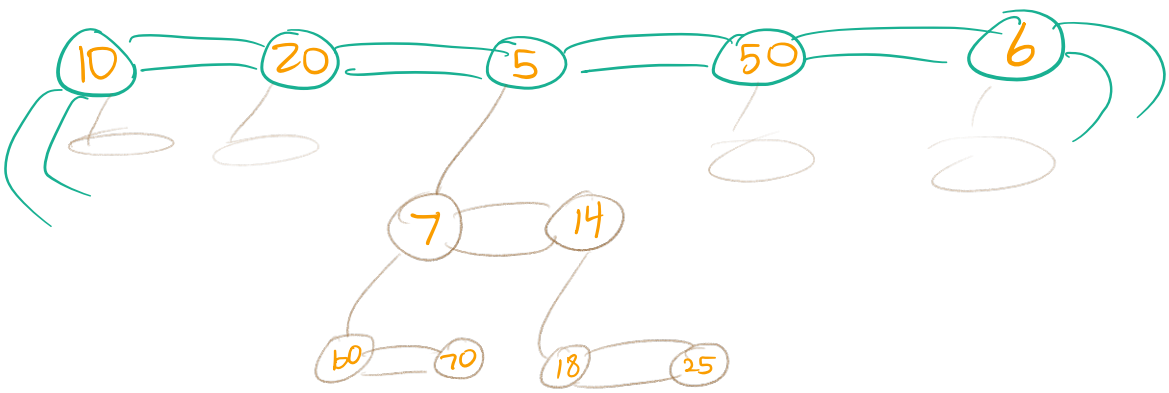


the root list is "healed" after extraction

- children of (5) ?

FibHeap.ExtractMin

```
Z = H → min
if  Z ≠ NULL
    for each child x of z
        add x to H's root list
        x→p = NULL
    remove z from rootlist of H
    if  Z == Z→ next
        H→min = NULL
    else H→ min = Z→ next
        CONSOLIDATE (H)
    H→n = H→n -1
return Z
```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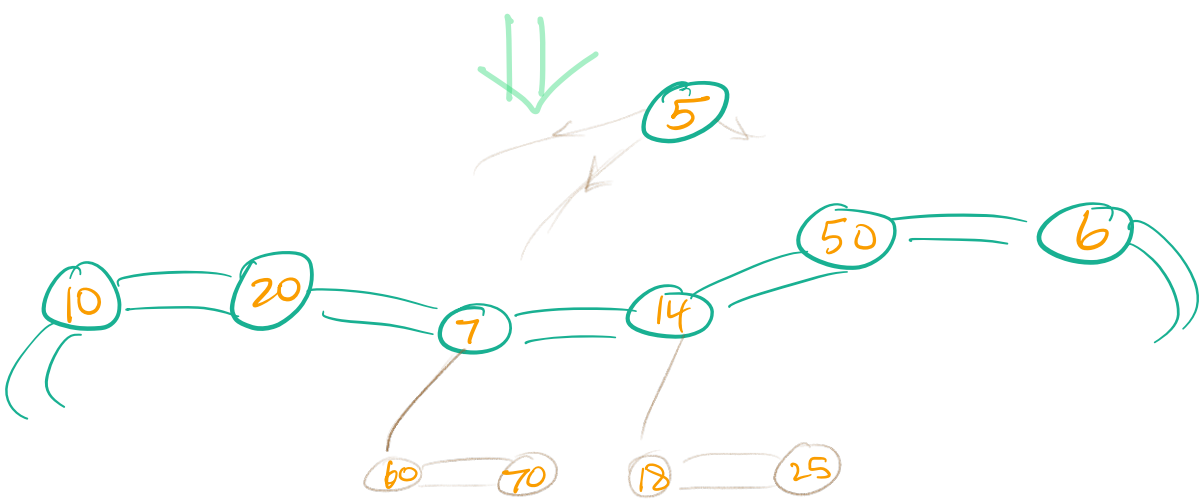
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