Computer Science 320 Midterm Test 2 NAME:______________________

Feb19 2015 Out of 48 marks

Recall that for a string w, $\#_\sigma(w)$ is the number of occurences of σ in w.

1. (2 mark) Either find, if they exist, a regular language L_1 and a context-free language L_2 whose intersection is regular, or explain why no such two languages exist.

2. (4 marks) True or False:

   (a) ___ The union of a (possibly infinite) number of regular languages can be non-regular

   (b) ___ If L_1 is regular and L_2 is context-free, then $L_1 \cup L_2$ is necessarily also context-free.

   (c) ___ Given a regular grammar G, each string in $L(G)$ has a unique derivation in G.

   (d) ___ If L is context-free, then so is L^*.

3. (6 marks) Give an algorithm to determine if a given Context-Free Grammar G is *usable*. Recall that a grammar is usable if there exists at least one string of terminals that can be derived from the start symbol of the grammar. In your answer, let the grammer be $G = (V, \Sigma, R, S)$, where
   S is the start symbol,
   Σ is the alphabet of terminals,
   V is set of variables, and
   R is the sets of production rules.
   You may refer to the *left side* of a rule, which consists of a single variable, and the *right side* of a rule, which consists of a string from $(V \cup \Sigma)^*$.

4. $L = \{w \in \{a, b\}^* :$ the latter half of the string contains only b's$\}$. An example of a string in the language is *abaababbbbbbb*. In other words, the last a appears in the first half of the string. In odd strings, the middle symbol must be b.

   (a) (4 marks) Give a natural PDA for the language given in part (a) – i.e., not a bottom-up or top-down parser obtained from your grammar.

   (b) (4 marks) Give a Context-Free Grammar for the language given above.

5. (4 marks) Give a Context-Free Grammar for the following: $\{a^{i+j}b^ic^j : i, j \geq 0\}$

6. (4 marks) Give a natural PDA for the above language – i.e., not a bottom-up or top-down parser obtained from your grammar.

7. (4 marks) Give a Context-Free Grammar for the following: $\{a^n b^m : 2n = 5m - 3$, and $n, m \geq 0\}$

8. (4 marks) Give a Context-Free Grammar for the following: $\{w \in \{a, b\}^* : w = xa^n b^n x^R$, where x is a string in $\{a, b\}^*\}$. Examples of such strings are: ab (where $x = \epsilon = x^R$), and $aababbaaabbabbabaa$ (where $x = aababba$ and $x^R = abbabaa$).

9. (4 marks) Give a natural PDA for the language in the previous question – i.e., not a bottom-up or top-down parser obtained from your grammar.

10. (4 marks) Give a Context-Free Grammar for the following: $\{w \in \{(,)\}^* : \#_((w) = \#_)(w)$ and every *suffix* of w has at least as many ('s as)'s$\}$. A suffix is the opposite of a prefix: for a string w, if w can be written as the concatenation of two strings, i.e., $w = w_1 w_2$, then the latter part, w_2 is a suffix of w. Note that ϵ is a suffix of every string, and w is a suffix of itself.

11. (4 marks) Give a bottom-up (Shift-Reduce) parser PDA for the following grammar.
$S \to X|Y$
$X \to Xc|A$
$A \to aAb|\epsilon$
$Y \to aY|B$
$B \to bBc|\epsilon$

12. **Bonus:** (3 bonus marks) Prove using the Pumping Lemma that the language L from question 6 is not regular; recall]
$L = \{w \in \{a, b\}^* :$ the latter half of the string contains only b's$\}$. An example of a string in the language is *abaababbbbbbb*. In other words, the last a appears in the first half of the string. In odd strings, the middle symbol must be b.