Computer Science 320 Test 1 – Feb 27 2025

NAME: Solutions 2026

Out of 68 marks 75 minutes

All questions to be answered on this exam. Extra space is available on the back page. Please indicate if you are using one of the back pages for your answer.

Notation:

- "$r_1 \equiv r_2$" means "regular expression r_1 is equivalent to (generates the same langauge as) regular expression r_2."
- "$11 \equiv_4 3$" means "11 is congruent to 3 mod 4" (i.e., 3 and 11 have the same remainder when divided by 4).
- The following languages have been studied and named within the course: Pal= $\{w \in \Sigma^* : w = w^R\}$ is the language of palindromes over Σ, where Σ is usually understood in the context; and $A^nB^n = \{a^nb^n | n \geq 0\}$.

1. (6 marks) Answer True or False, by placing a T or F beside the statement. Answer on this sheet. **Alert:** Each is worth one mark. Don't spend too much time on them until you have worked on the rest of the test.

   T If L is any regular language, then L^* is also regular.

   T There is a FA that accepts the language $\{w \in \{a,b\}^* : \#_a(w) \equiv_7 4\}$

   T $(ab^* + ba^*)^* \equiv b^*(a+b)^*$

   F $\emptyset^* \cdot (a+b)^* \equiv ((a+b)\cdot\emptyset)^*$

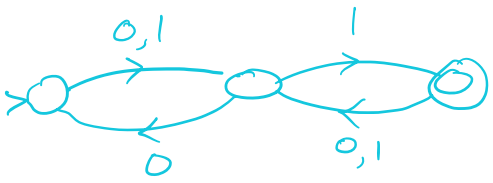
   F Every string in $a(a+b)b^* + b(ab+bab)^*$ ends in b.

   T $((a+\epsilon)(b+\epsilon))^* \equiv (a+b)^*$

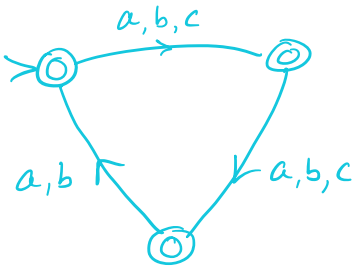
2. (4 marks) Give a regular expression for the strings over alphabet $\{a,b,c,d\}$ such that each c is followed, eventually, by a d. E.g., *acdaccacbbbdaaaa* is in the language, but *cacadcab* is not. (It is the same as saying, "The rightmost d is right of the rightmost c.")

   $((a+b+c+d)^*d + \epsilon)\cdot(a+b+d)^*$

3. (4 marks) Give a FA (nondeterminism is okay but not needed) that accepts the language of all binary strings (i.e., strings over $\{0, 1\}$) that have even length and end in 1.



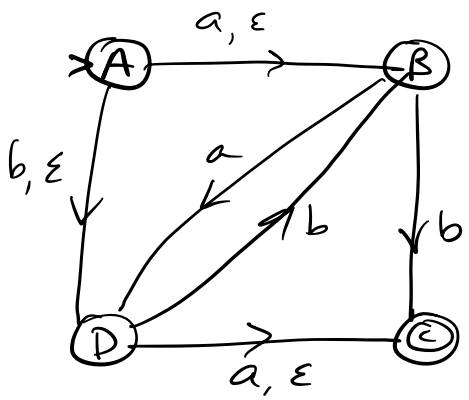
4. (4 marks) Give a DFA that accepts the language $\{w \in \{a, b, c\}^* :$ every third letter in w is either a or $b\}$. This means the third letter, if it exists, cannot be c, nor can the sixth, etc. For example, *caacba* is in the language, but *abccca* is not. Examples of strings in the language are $\epsilon, a, b, c, cc, ccac, aaac$, etc.



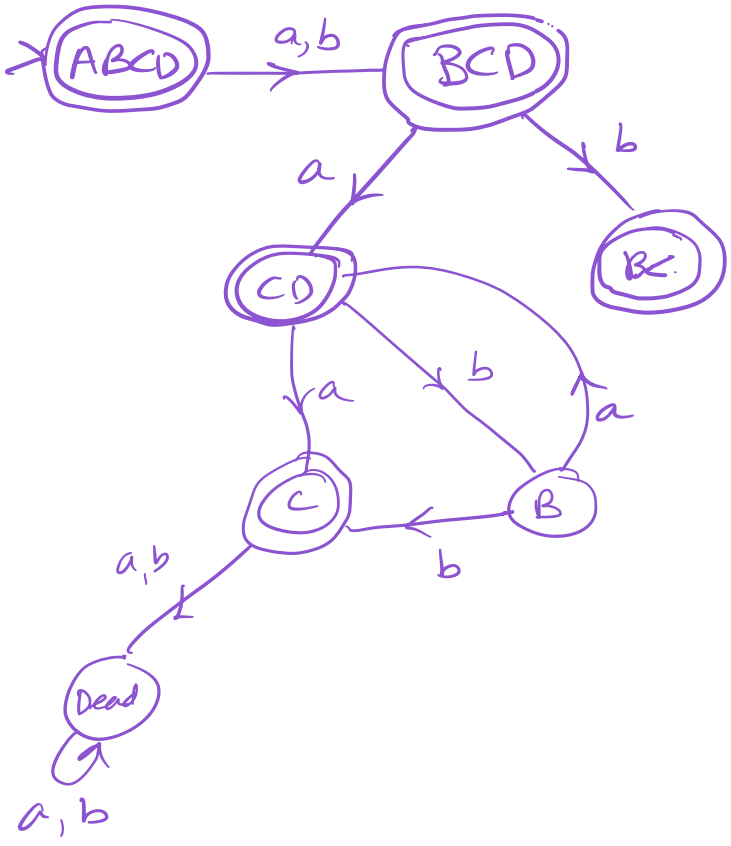
5. (4 marks) Give a regular expression for the language in the previous question. For full marks, make the regular expression reasonably concise.

$$\Big((a+b+c)(a+b+c)(a+b)\Big)^*(\varepsilon+a+b+c)(\varepsilon+a+b+c)$$

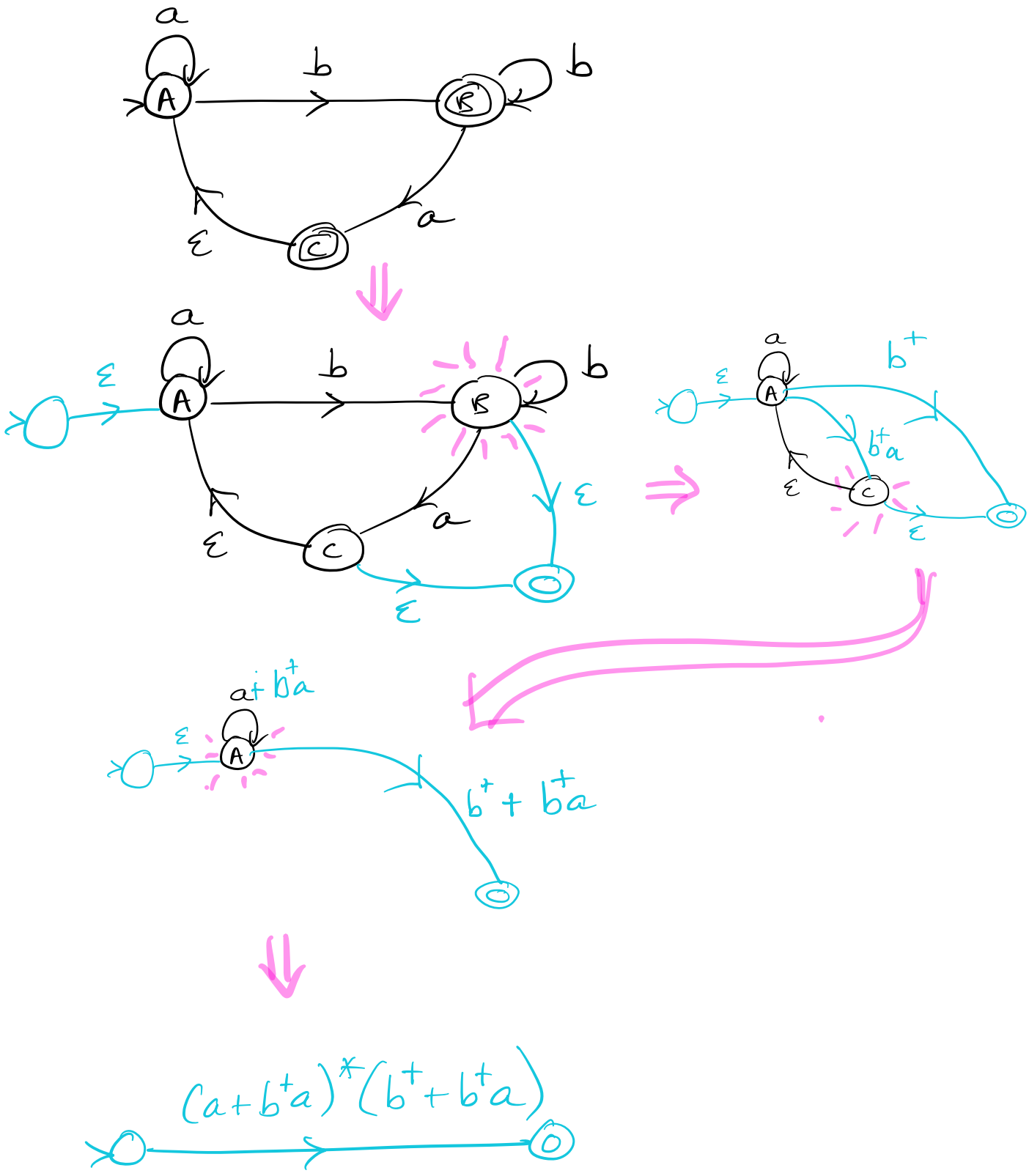
6. (8 marks) For the following NFA, use the construction (algorithm) to devise an equivalent DFA.



	a	b
A	B	CD
B	CD	C
C	∅	∅
D	C	B
> ABCD	BCD	BCD
BCD	CD	BC
CD	C	B



7. (8 marks) Use the construction (algorithm) to devise a regular expression for the language accepted by the following NFA. Rip the states in this order: 2, 3, 1.



8. (8 marks) Use the construction (algorithm) to build a NFA that accepts the language

$$b(a^* + c) + (bb)^*$$

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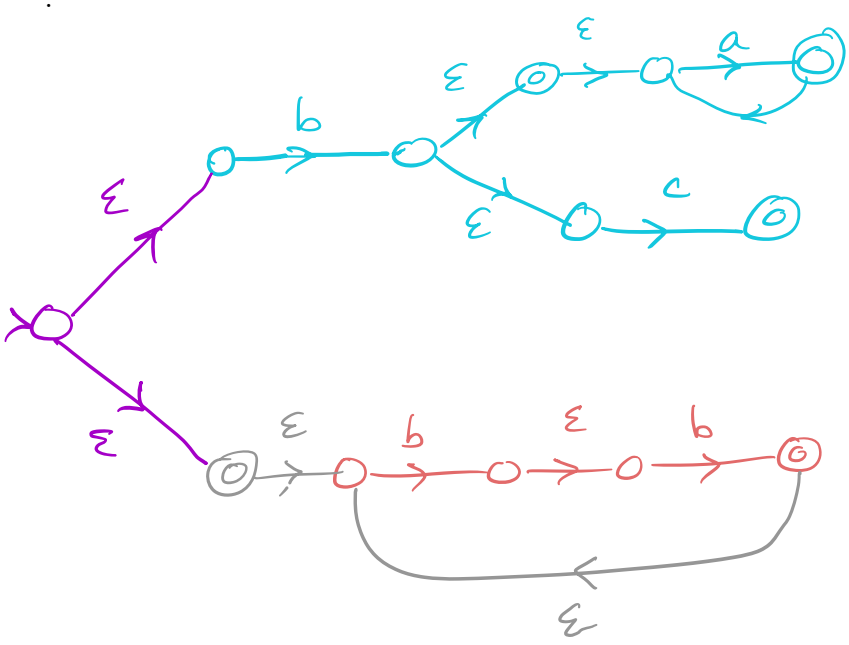


9. (4 marks) Give a Context-Free Grammar for the language $\{a^n b^m a^{m+n} | n, m \geq 0\}$

$S \to aSa \mid B$

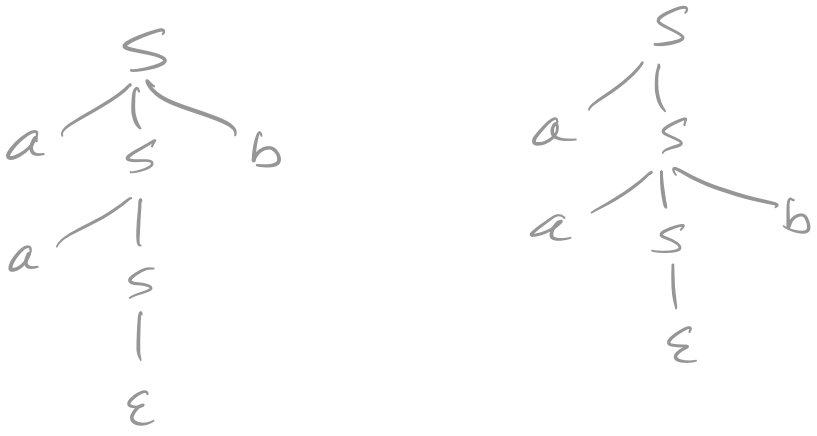
$B \to aBb \mid \varepsilon$

10. (2 marks) What language does $S \to \ aSb \ \mid aS \mid \epsilon$ generate? Give a formal description.

$\{ a^i b^j \mid i \geq j \geq 0 \}$

11. (4 marks) Is the following grammar ambiguous? Prove it or argue to the contrary. (Recall: a grammar is ambiguous if some string has two distinct derivation trees/leftmost derivations.)
$S \to \ aSb \ \mid aS \mid \epsilon$

Two derivations of aab



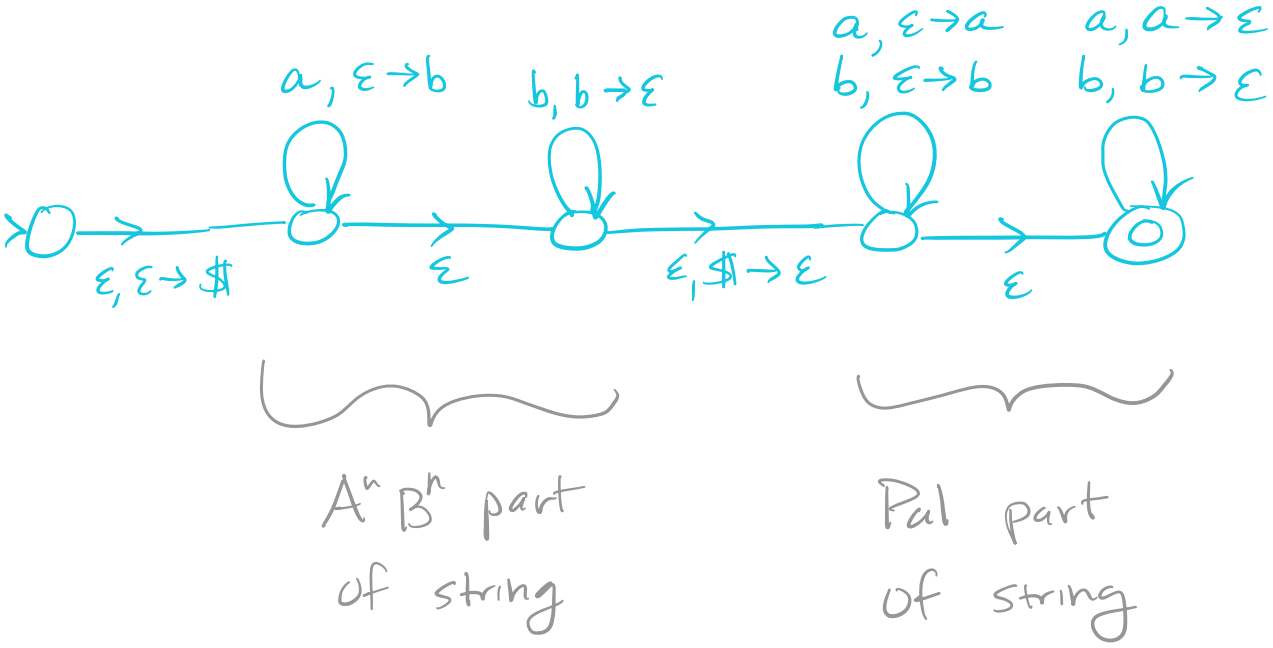
12. (4 marks) Give a Context-Free Grammar for the language $\{a^n b^n w \mid n \geq 0, w \in \{a,b\}^*$ and w is a palindrome$\}$. Note that this language is the concatenation of two languages, $A^n B^n$ and Pal. Some strings in the language are *abba* and *aabbabba*

$S \to NP$

$N \to aNb \mid \varepsilon$

$P \to aPa \mid bPb \mid \varepsilon \mid a \mid b$

(8 marks) Give a natural PDA for the language of the previous question, i.e. for $\{a^n b^n w \mid n \geq 0, w \in \{a,b\}^*$ and w is a palindrome$\}$. [Hint: a bottom of stack marker may be useful for when passing from the $A^n B^n$ part of the string to the Pal part of the string.] For part marks, the student may instead give two PDA's, one for $A^n B^n$ and one for Pal.



The bottom-of-stack marker ensures that the b's from the $A^n B^n$ part do not get mistakenly interpretted as part of the palindrome.

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