Tutorial - Enumerating + Diagonalization.

March 3, 2026

For each set below, either prove it is uncountable or that it is countable.

1. $\mathbb{Z}^- = \{ n \mid n \in \mathbb{Z}, n < 0 \}$

2. $\{ -3, -2, -1, 0, 1, 2, 3 \}$

3. $L(a(a+b)^* a)$

4. $\mathbb{W} \times \mathbb{W}$, $\mathbb{W} = \{ 0, 1, 2, 3, \ldots \}$

5. The set of all languages over $\{a, b\}$

6. The set of all regular languages over $\{a, b\}$

7. The set of even numbers.

8. The union of n countably infinite sets, where n is a finite integer.

9. The union of a countably infinite number of countably infinite sets.

4. $\mathbb{W} \times \mathbb{W}$ is countably infinite, as demonstrated by the following enumeration...

[Recall $\mathbb{W} \times \mathbb{W} = \{(0,0), (0,1), (1,0), (1,1), (0,2), \ldots\}$]