Tutorial Week 1 Jan 6, 2023

From chapter 0 ...

Sets are like this : $\{a, b\}$

curly bracket

does not imply an order

If there is an (implied) "universe" $\mathcal{U}$, or ground set from which the set is drawn, then set complementation is defined

$$\overline{S} = U \setminus S$$

"S complement"

all elements not in S

"set.minus"

We also have $\cap$, $\cup$

Sequences and tuples - order is implied

- use ( )

"open paren" "close paren"

(Note : graph theory, undirected graphs, abuse this notation somewhat, using (u,v) for an undirected edge.)

Review: - Functions + relations
- Graphs.

STRINGS & LANGUAGES

An alphabet is a finite, non-empty set.
Its members are called symbols (or letters).

Σ "capital sigma"
Γ "capital gamma"
} often used to denote alphabets.

A string over an alphabet Σ is a finite sequence of symbols from Σ, duplicates allowed.

ε ("epsilon") is the symbol for the empty string.

$ab \cdot bb = abbb$
$s_1 \cdot s_2 = s_1 s_2$

Juxtaposing two strings is an operation called "concatenation", yielding a new string. Also use $\bullet$

Recursive Definitions.

Alternative definition of strings over $\{a,b\}$.

Defn: 1. ε is a string over $\{a, b\}$

2. if s is a string over $\{a, b\}$, so is $s \cdot a$

3. if s is a string over $\{a, b\}$, so is $s \cdot b$

4. Nothing else is a string over $\{a, b\}$

aab a string over $\{a, b\}$?

The above is a recursive definition of strings (over the given alphabet).

Once we have a recursive definition of an object, we can easily define functions of that object.

Def$^{\underline{n}}$ length of a string over $\{a,b\}$, denoted with abs-value $|s|$

1. $|\varepsilon| = 0$
2. $|s \cdot a| = |s| + 1$, where s is a string over $\{a,b\}$
3. $|s \cdot b| = |s| + 1$ " " "

Length is now defined for all strings over $\{a,b\}$, since (4) "Nothing else is a string over $\{a,b\}$"

Def^n string over Σ.

1. ε is a string over Σ
2. if s is a string over Σ and $\sigma \in \Sigma$ then $s \cdot \sigma$ is a string over Σ
3. Nothing else is a string over Σ.

For you to do:

1. Give a definition of string length for strings over Σ.

2. We want $\#_a(s)$ defined on strings over $\{a, b\}$ to be the number of a's in s.
Make it so, using the recursive definition.

3. Do same for $\#_\sigma(s)$ for strings over Σ, $\sigma \in \Sigma$.

4. (challenge) give a coherent recursive definition of concat $(\cdot)$ for strings over Σ

5. Problem 0.12
Claim: For any set S of horses, $|S| = \underline{n} \geq 1$, the horses in the set are the same

colour. (monochromatic)

"Proof": By induction on n.

Basis: $n = 1$. Obvious.

Induction Step: S $|S| = K > 1$

Ind Hyp: $\forall\, K' < K$, any set of K' horses is monochromatic.

Take one horse (first hor) out of S, yielding S'.

$|S'| < K$ so S' is monochromatic, by IH.

Put that horse back in and take out a different horse (horse 2), yielding S''.

S'' horses are also all same colour as first horse, by Ind Hyp.

$\therefore$ all horses in S are same colour. ∎

What is wrong?

1 _ 3 4 h_1 5 . . .

h_1 B h_2 W

Defn of string length for strings over Σ.

1. $|\varepsilon| = 0$
2. $|s \cdot \sigma| = |s| + 1$ $\forall$ strings s over Σ, $\forall\, \sigma \in \Sigma$.

Defn of $\#_a(s)$, s a string over $\{a,b\}$.

1. $\#_a(\varepsilon) = 0$
2. $\#_a(s \cdot a) = \#_a(s) + 1$
3. $\#_a(s \cdot b) = \#_a(s)$

Defn of $\#_\sigma$, a function of strings over Σ. $\sigma \in \Sigma$.

1. $\#_\sigma(\varepsilon) = 0$
2. $\#_\sigma(s \cdot \sigma) = \#_\sigma(s) + 1$
3. $\#_\sigma(s \cdot \gamma) = \#_\sigma(s)$ $\gamma \neq \sigma$, $\gamma \in \Sigma$.