# Decidability, Recognizability, Undecidability.

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Prove true claims (decidable? undecidable? recognizable? unrecognizable?) about the following languages:

0. $H_{TM} = \{ \langle M, w \rangle \mid M$ is a TM that halts on string $w \}$

1. $A_{TM} = \{ \langle M, w \rangle \mid M$ is a TM that accepts string $w \}$

2. $NonE_{DFA} = \{ \langle D \rangle \mid D$ is a DFA and $D$ accepts some string $\}$

3. $AccE_{PDA} = \{ \langle P \rangle \mid P$ is a PDA and $P$ accepts $\varepsilon \}$

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* 4. $RejectsSomeString_{TM} = \{ \langle M \rangle \mid M$ is a TM and $M$ rejects some string $\}$.

* 5. $Loops_{TM} = \{ \langle M, w \rangle \mid M$ is a TM that loops on input $w \}$

* 6. $SixStates_{TM} = \{ \langle M \rangle \mid M$ is a TM and $M$ has $\geq 6$ states $\}$.

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$NonE_{DFA} = \{ \langle D \rangle \mid D$ is a DFA and $L(D) \neq \emptyset \}$

Claim: $NonE_{DFA}$ is

Proof: Recall we showed that $E_{DFA} = \{ \langle D \rangle \mid D$ is a DFA, $L(D) = \emptyset \}$ is decidable; let $X$ be a decider-TM for $E_{DFA}$.

We construct a TM $Y$ that decides $NonE_{DFA}$ as follows:

$Y$ = " on input $\langle D \rangle$ where $D$ is a DFA:

1. Run on input $\langle D \rangle$.
   - if $X$ accepts, REJECT.
   - if $X$ rejects, ACCEPT."

$Y$ halts, because $X$ is a decider and so always halts.

$Y$ is correct, because $\langle D \rangle \in NonE_{DFA}$ iff $\langle D \rangle \notin E_{DFA}$.

$\therefore$ $Y$ decides $NonE_{DFA}$. $\blacksquare$