CSCI 320 Practice Midterm II NAME:______________________

Note that we use "TM" to mean "Turing Machine". The following languages are known to be **Undecidable**
$A_{TM} = \{< M, w > \mid M$ is a TM that accepts string $w\}$.
$Halt_{TM} = \{< M, w > \mid M$ is a TM that halts on string $w\}$.
$E_{TM} = \{< M > \mid M$ is a TM that accepts no strings$\}$.
$EQ_{TM} = \{< M_1, M_2 > \mid M_1$ and M_2 are TMs and $L(M_1) = L(M_2)\}$.

1. (10 marks) Show the the following language is **undecidable**:
   $OneAccepts_{TM} = \{< M_1, M_2, w > |M_1$ and M_2 are TMs exactly one of them accepts $w\}$. Do not use Rice's Theorem. For full marks, give a brief argument that shows your reduction works.

2. (a) (4 marks) Prove that the class of Turing-decidable languages is closed under intersection. [Hint: "Let L_1 and L_2 be two Turing-decidable languages, decided by X_1 and X_2 respectively. Then we can construct a TM X that decides the language $L_1 \cap L_2$ as follows: ..."]
   (b) (6 marks) Prove that the class of Turing-recognizable languages is closed under union.

3. Are the following languages recognizable? Prove your answer.
   (a) (10 marks) Reject$_{any} = \{< M > |M$ is a TM, and there is a string w that M rejects $\}$.
   (b) (10 marks) OneAccepts$= \{< M_1, M_2, w > |M_1$ and M_2 are TMs, and exactly one of M_1 and M_2 accepts $w.\}$.