Computer Science 320 Midterm Test II NAME: Solutions
March 17 2016. Out of 51 marks

You may refer to the following languages by name:
A^nB^n is the language $\{a^nb^n : n \in \mathbf{N}\}$.
Prime is a language over a unary alphabet $\{\alpha\}$ where $\alpha^p \in$ Prime iff p is a prime number.
Evenpal over Σ is the language defined as $\{ww^R : w \in \Sigma^*\}$.
Pal over Σ is the language defined as $\{w : w = w^R \in \Sigma^*\}$.
(Recall that for a string w, w^R is the reverse of w, and $L^R = \{w : w^R \in L\}$.)

1. (2 mark) Either find, if they exist, two non-regular Context-Free languages L_1, L_2 whose intersection is regular, or explain why no such two CF languages exist.

$L_1 = \{a^nb^n \mid n \geq 0\}$ $L_2 = \{b^na^n \mid n \geq 0\}$

Both are CF but not regular, and $L_1 \cap L_2 = \{\varepsilon\}$, which is regular.

2. (6 marks) True or False:

   (a) F There is a deterministic PDA that accepts the language of palindromes over alphabet $\Sigma = \{a, b\}$.

   (b) F Given a context-free grammar G, each string in $L(G)$ has a unique derivation in G.

   (c) F It is always the case that if L is context-free, then so is Ls complement $\overline{L}$.

3. (5 marks) Is the class of Context-Free languages closed under reverse? Prove your answer. (Hint: If a language is CF then it has a CF Grammar.)

Yes. Convert a CFG for L to a CFG for L^R by reversing the RHS of every rule in the grammar.

Eg $S \to aSc \mid X$
$X \to bXc \mid \varepsilon$
$\Rightarrow$
$S \to cSa \mid X$
$X \to cXb \mid \varepsilon$

4. (6 marks) Show how to construct a CF grammar for $L_1 \cup L_2$, given that L_1 has the grammar (V_1, Σ, R_1, S_1) and L_2 has the grammar (V_2, Σ, R_2, S_2). You can assume that V_1 and V_2 are disjoint (their intersection is empty).

A grammar for $L_1 \cup L_2$ is

$(V_1 \cup V_2 \cup \{S\}, \Sigma, R_1 \cup R_2 \cup \{S \to R_1, S \to R_2\}, S)$

5. (12 marks) Give a context-free grammar for each of the following languages:

   (a) $\{a^{i+j+k} b^i c^j d^k : i, j, k \geq 0\}$

$S \to aSd \mid C$

$C \to aCc \mid B$

$B \to aBb \mid \varepsilon$

(b) $\{w \in \{(,),[,]\}^* : w$ is a balanced string of both kinds of parentheses, and square brackets can appear within rounded parentheses but rounded parentheses cannot appear within square brackets $\}$. E.g., ([])[]() is a sting in the language, but [()] is not, nor is [(]).

$S \to (S) \mid SS \mid X$

$X \to [X] \mid XX \mid \varepsilon$

(c) $\{w \in \{0,1,\$\}^* : w$ contains exactly one \$, and has more 0's than 1's before the \$, and more 1's than 0's after the \$ $\}$

$S \to A0A\$B1B$ // $Z \Rightarrow^*$ strings with same num 0's as 1's

$A \to A0 \mid AA \mid Z$ // $A \Rightarrow^*$ strings with at least as many 0's as 1's

$B \to B1 \mid BB \mid Z$ // $B \Rightarrow^*$ strings with at least as many 1's as 0's

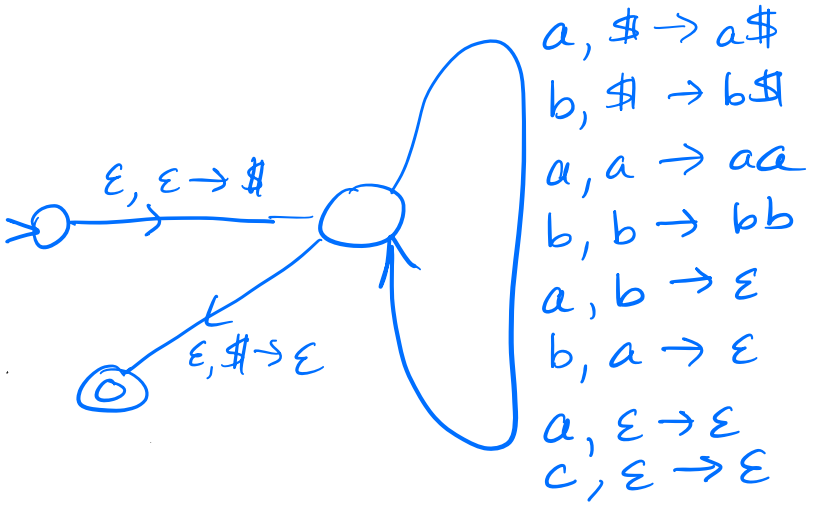
$Z \to Z0Z1Z \mid \varepsilon$

6. (4 marks) What language is generated by the following grammar?
$S \to X|SS$
$X \to aXc|B$
$B \to Bb|\epsilon$

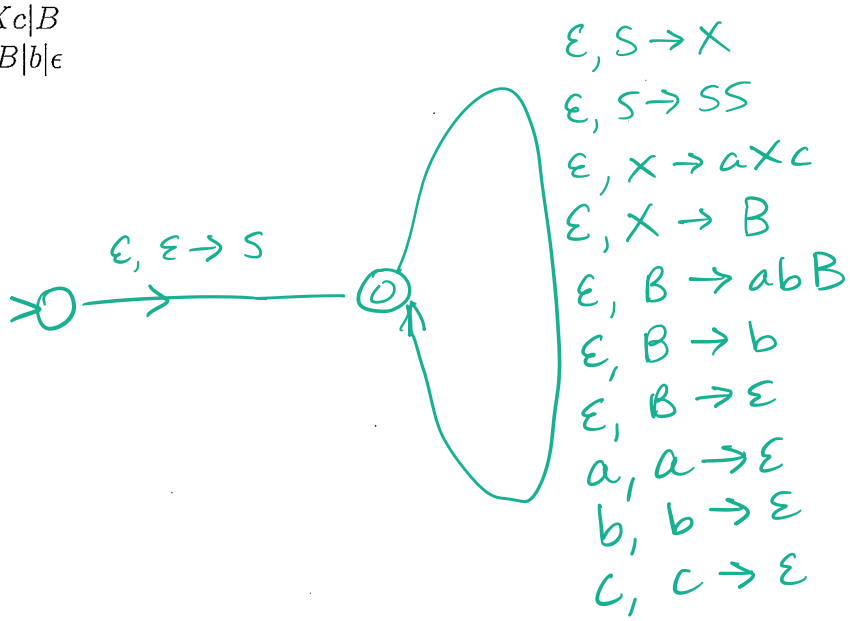
Give a precise description in English, or a precise mathematical description of the language.

$\{ w \in \{a,b,c\}^* \mid$ w is a balanced string of parens with "a" subbed for "(" and "c" subbed for ")", and where every block of b's is immediately preceded by "a" and immediately succeeded by "c" $\}$

7. (5 marks) Give a natural PDA for the language $L = \{w \in \{a, b, c, \}^* : \#_b(w) \leq \#_a(w)\}$.



8. (5 marks) Give a Top-Down Parser PDA for the following CF Grammar:
$S \to X|SS$
$X \to aXc|B$
$B \to abB|b|\epsilon$



9. (a) (3 marks) Give an example of a non-regular Context-Free language and an infinite regular language such that their intersection is regular. Give a precise description of the language that is the intersection of the two.

$A^n B^n$.

$L_2 = L(b^* a^*)$

$A^n B^n \cap L_2 = \{\varepsilon\}$

(b) (3 marks) Give an example of a Context-Free language and a Regular language such that their intersection is not regular. Give a precise description of the language that is the intersection of the two.

$A^n B^n$ is CF

$L_2 = L((a+b)^*)$ is regular

$A^n B^n \cap L_2 = A^n B^n$, which is not regular.