Computer Science 320 Midterm Test 1 NAME: Solutions

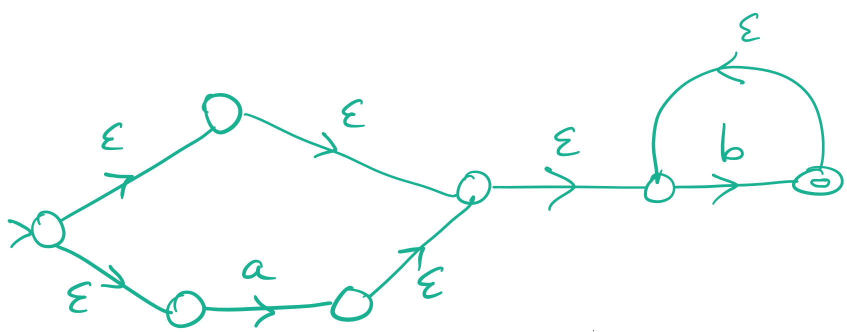
Jan 22 2015 Out of 57 marks

Recall that for a string w, $\#_\sigma(w)$ is the number of occurences of σ in w.

1. (1 mark) What is the closure of the set $\{6, 8\}$ under subtraction?

$\{n \in \mathbb{Z} : n \text{ is even}\}$

2. (4 marks) **Using the construction,** give the NFA that corresponds to the regular expression $(\epsilon + a)b^*$. Do not include a dead state. Do include all ϵ transitions that the construction dictates (even if they are not useful).



3. (10 marks) For each pair of regular expressions below, do they represent the same language? Answer True or False. If False, give a string that differentiates them (i.e., is in the language of one but not the other).

(a) aa^* and $(a + aa)^*$ F. ε

(b) $(ab^+ + b)^*$ and $(a^*b^*)^*$ F. a

(c) $a^* + b^*$ and $(a + b)^*$ F. ab

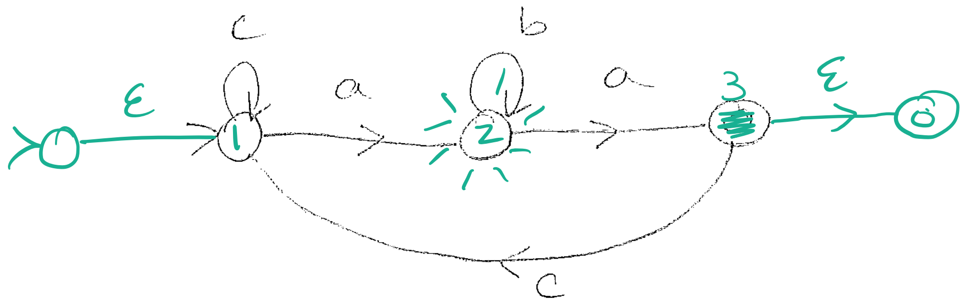
(d) $(a + b)^*ab(a + b)*$ and $(a^+b^+)^*$ F. ε

(e) $((\epsilon + a)^*b)^*$ and $(a + b)^*b$ F. ε

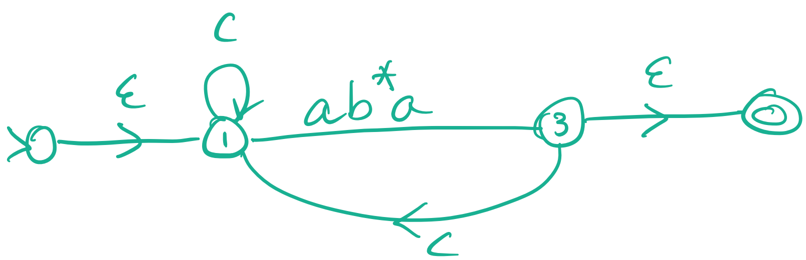
4. (4 marks) Give a **regular expression** for the language of non-empty strings over $\{a, b\}$ that begin and end in the same letter and have a number of b's that is evenly divisible by 2.

$a(a^*ba^*ba^*)^*a + b(a^*ba^*ba^*)^*b$

6. (5 marks) **Use the construction** to find the RE that describes the language accepted by the following FA.



Rip 2:



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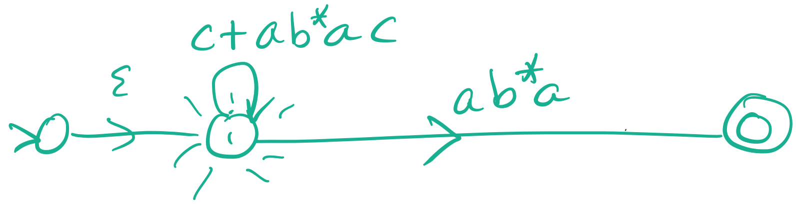
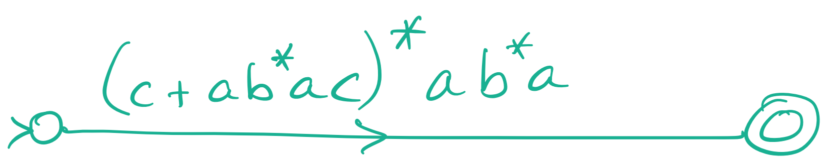
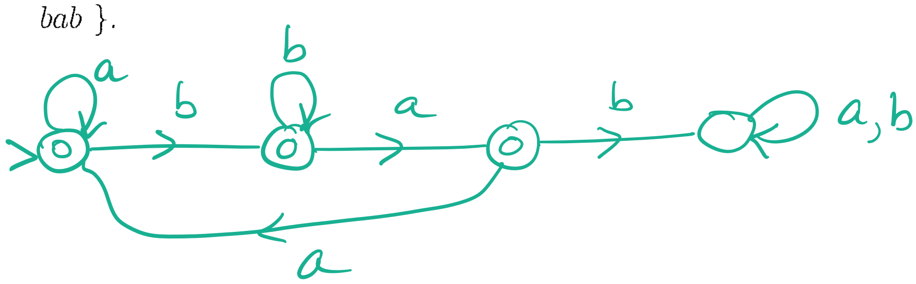


Rip 1:



(b) Give a DFA for the language $L = \{w \in \{a,b\}^* : w$ does not contain the substring $bab\}$.



(c) (4 marks) Let M_1 and M_2 be two DFA's, where

$$M_1 = \{Q_1, \Sigma, \delta_1, q_1, F_1\}$$

$$M_2 = \{Q_2, \Sigma, \delta_2, q_2, F_2\}$$

Show how to contruct the DFA $M' = \{Q', \Sigma, \delta', q', F'\}$ for the language $L(M_1) \cap L(M_2)$. That is, show the construction for the intersection of regular languages. Do so by giving the following:

Q' : $Q_1 \times Q_2$

q' : (q_1, q_2)

F' : $F_1 \times F_2$

δ' : $\delta'((q,p),\sigma) = (\delta_1(q,\sigma), \delta_2(p,\sigma))$

$\forall\ q \in Q_1,\ p \in Q_2,\ \sigma \in \Sigma$

You are encouraged to use the mathematical notation given in class, but it is also acceptable to describe the sets, ordered pairs, or functions above in precise English. (You may wish to do part (d) below first, as a model for your general solution.)

Computer Science 320 PL Test NAME:____________________

Mar 1, 2016 50 minutes Out of 20 marks

1. (8 marks) Prove, using the Pumping Lemma for Regular Languages, that the language $L = \{xx : x \in \{a, b\}^*\}$ is not regular. (To understand the language L: for example, $\epsilon \in L$, since $\epsilon = \epsilon \cdot \epsilon$; however, the string aabbaa is not in L because the first half of the sting, aab, does not match the last half of the string, baa.)

2. (8 marks) Let L be the language $\{w \in \{a, b, c\}^* : w$ contains at least two occurences of the letter c, and the substring between the first c and the second c contains a and b in equal number.$\}$ An example of a string in the language L is *aabcaababbcbbbbbacacc*. Show using the Pumping Lemma that L is not regular. It is recommended you use the closure theorems.

3. (4 marks) For this question, you may refer to the following languages by name, and may assume that they are non-regular:
A^nB^n is the language $\{a^nb^n : n \in \{0, 1, 2, ...\}\}$.
Evenpal over Σ is the language defined as $\{ww^R : w \in \Sigma^*\}$.
Pal over Σ is the language defined as $\{w : w = w^R \in \Sigma^*\}$.
Show, using the closure theorems and the fact that the above-mentioned languages are not regular, that the language L is not regular, where $L = \{w \in \{a, b, c\}^* : \#_a(w) + \#_c(w) = \#_b(w)\}$.