## .7 Summary of D, SD/D and ¬SD Languages that Include Turing Machine Descriptions

At the beginning of this chapter, we presented a table with a set of questions that we might like to ask about Turing machines and we showed the language formulation of each question. We have now proven where most of those languages fall in the D, SD/D, ¬SD hierarchy that we have defined. (The rest are given as exercises.) So we know whether there exists a decision procedure, a semidecision procedure, or neither, to answer the corresponding question. Because many of these questions are very important as we try to understand the power of the Turing machine formalism, we summarize in Table 21.2 the status of those initial questions, along with some of the others that we have considered in this chapter.

**Table 21.2** The problem and the language view.

| *The Problem View* | *The Language View* | *Status* |
|---|---|---|
| Does TM $M$ have an even number of states? | $\{<M>:$ TM $M$ has an even number of states$\}$ | D |
| Does TM $M$ halt on $w$? | $H = \{<M, w>:$ TM $M$ halts on $w\}$ | SD/D |
| Does TM $M$ halt on the empty tape? | $H_\varepsilon = \{<M>:$ TM $M$ halts on $\varepsilon\}$ | SD/D |
| Is there any string on which TM $M$ halts? | $H_{ANY} = \{<M>:$ there exists at least one string on which TM $M$ halts $\}$ | SD/D |
| Does TM $M$ halt on all strings? | $H_{ALL} = \{<M>:$ TM $M$ halts on $\Sigma^*\}$ | ¬SD |
| Does TM $M$ accept $w$? | $A = \{<M, w>:$ TM $M$ accepts $w\}$ | SD/D |
| Does TM $M$ accept $\varepsilon$? | $A_\varepsilon = \{<M>:$ TM $M$ accepts $\varepsilon\}$ | SD/D |
| Is there any string that TM $M$ accepts? | $A_{ANY}\ \{<M>:$ there exists at least one string that TM $M$ accepts $\}$ | SD/D |
| Does TM $M$ fail to halt on $w$? | $\neg H = \{<M, w>:$ TM $M$ does not halt on $w\}$ | ¬SD |
| Does TM $M$ accept all strings? | $A_{ALL} = \{<M>: L(M) = \Sigma^*\}$ | ¬SD |
| Do TMs $M_a$ and $M_b$, accept the same languages? | $EqTMs = \{<M_a, M_b>: L(M_a) = L(M_b)\}$ | ¬SD |
| Is it the case that TM $M$ does not halt on any string? | $H_{\neg ANY} = \{<M>:$ there does not exist any string on which TM $M$ halts$\}$ | ¬SD |



## Exercises

1. For each of the following languages $L$, state whether $L$ is in D, SD/D, or ¬SD. Prove your claim. Assume that any input of the form $<M>$ is a description of a Turing machine.

   **a.** $\{\texttt{a}\}$
   **b.** $\{<M>: \texttt{a} \in L(M)\}$
   **c.** $\{<M>: L(M) = \{\texttt{a}\}\}$
   **d.** $\{<M_a, M_b>: M_a$ and $M_b$ are Turing machines and $\varepsilon \in L(M_a) - L(M_b)\}$
   **e.** $\{<M_a, M_b>: M_a$ and $M_b$ are Turing machines and $L(M_a) = L(M_b) - \{\varepsilon\}\}$
   **f.** $\{<M_a, M_b>: M_a$ and $M_b$ are Turing machines and $L(M_a) \neq L(M_b)\}$
   **g.** $\{<M, w>: M$, when operating on input $w$, never moves to the right on two consecutive moves$\}$
   **h.** $\{<M>: M$ is the only Turing machine that accepts $L(M)\}$
   **i.** $\{<M>: L(M)$ contains at least two strings$\}$
   **j.** $\{<M>: M$ rejects at least two even length strings$\}$
   **k.** $\{<M>: M$ halts on all palindromes$\}$
   **l.** $\{<M>: L(M)$ is context-free$\}$
   **m.** $\{<M>: L(M)$ is not context-free$\}$
   **n.** $\{<M>: A_{\#}(L(M)) > 0\}$, where $A_{\#}(L) = |L \cap \{\texttt{a}^*\}|$
   **o.** $\{<M>: |L(M)|$ is a prime integer $> 0\}$
   **p.** $\{<M>:$ there exists a string $w$ such that $|w| < |<M>|$ and that $M$ accepts $w\}$
   **q.** $\{<M>: M$ does not accept any string that ends with $0\}$
   **r.** $\{<M>:$ there are at least two strings $w$ and $x$ such that $M$ halts on both $w$ and $x$ within some number of steps $s$, and $s < 1000$ and $s$ is prime$\}$
   **s.** $\{<M>:$ there exists an input on which $M$ halts in fewer than $|<M>|$ steps$\}$
   **t.** $\{<M>: L(M)$ is infinite$\}$
   **u.** $\{<M>: L(M)$ is uncountably infinite$\}$
   **v.** $\{<M>: M$ accepts the string $<M, M>$ and does not accept the string $<M>\}$
   **w.** $\{<M>: M$ accepts at least two strings of different lengths$\}$
   **x.** $\{<M>: M$ accepts exactly two strings and they are of different lengths$\}$