5. "Closure" and "Closed"

$\mathbb{N} = \{1, 2, 3, \ldots\}$

$W = \{0, 1, 2, \ldots\}$

$\mathbb{Z} = \{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}$

$\mathbb{Q}^+ = \{\frac{a}{b} : a, b \in \mathbb{N}\}$

$\mathbb{Q} = \{\frac{a}{b} : a \in \mathbb{Z}, b \neq 0 \text{ and } b \in \mathbb{Z}\}$.

Defn: A set S is "closed under" an operation $\blacksquare$ if

$$\forall a, b \in S, \quad a \blacksquare b \in S$$

(supposing $\blacksquare$ to be a <u>binary</u> operation)

Eg: Is $\mathbb{N}$ closed under $+$?
Is $\mathbb{N}$ closed under $-$?

Is W closed $+$? $-$?

Is $\mathbb{Q}^+$ closed under "recip" function

recip$(x) = \frac{1}{x}$?

(we can extend "closed under" to unary ops)

Is $\mathbb{Q}$ closed under negation?

Is $\mathbb{Z}$ closed under negation?

Defn: For a set S, the "closure of under $\blacksquare$" is the set

$$S \cup \{ x \blacksquare y : x, y \in S \}$$

Eg the closure of $\mathbb{N}$ under "$+$" is $\mathbb{N}$

the closure of $\mathbb{N}$ under "$-$" is $\mathbb{Z}$

the closure of $\mathbb{N}$ under "$\div$" is $\mathbb{Q}^+$

Consider an alphabet $\Sigma = \{a, b, c\}$.

What is the closure of Σ under concat "$\cdot$"?