Chapter 1 Regular Languages

From Chapter 0:

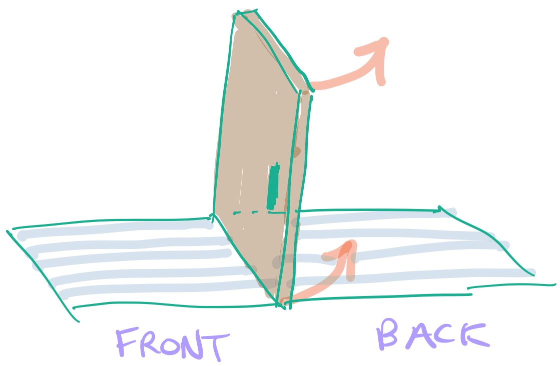
$\Sigma = \{a, b\}$ Alphabet

abba string

$\{a, aa, aaa, aba, aaaa,$
$aaba, abaa, abba, aaaaa,$
$\ldots\}$ Language

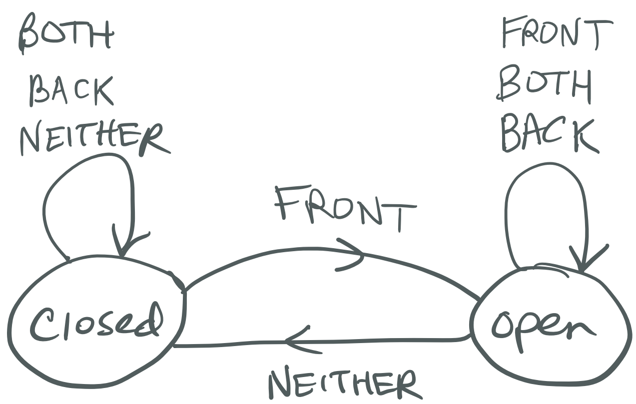
Def$\underline{n}$: A language over alphabet Σ is a set of strings over Σ.

Finite Automata



What behavior do we want from an automatic door?

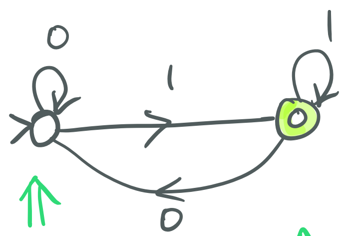
— if FRONT is occupied ("FRONT")
then swing open ...
unless BACK is also occupied.
... and assuming door is closed.
⋮



Transition Table for automatic door controller:

	NEITHER	FRONT	REAR	BOTH
CLOSED	CLOSED	OPEN	CLOSED	CLOSED
OPEN	CLOSED	OPEN	OPEN	OPEN

More examples of FAs:

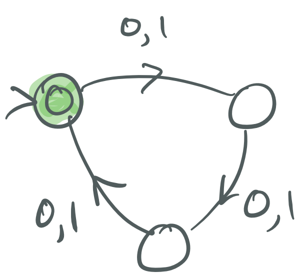


start state

"Final" or "Accept" state

⇐ what kind of strings can drive the FA from the start state to end at a final state

" [redacted] "



⇐ string 010 is accepted

string 0101 is not...

it is rejected.

" strings over {0,1} of length that is a mult of 3".

Def$^{\underline{n}}$: A **finite automaton** is a 5-tuple $(Q, \Sigma, \delta, q_0, F)$ where:

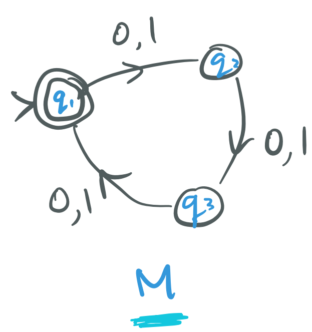
Q is a finite set of **states**

Σ is an **alphabet**

$\delta : Q \times \Sigma \to Q$ is a **transition function**

$q_0 \in Q$ is the **start state**

$F \subseteq Q$ is the set of **accept states**

Eg: 

$M = (\{q_1, q_2, q_3\}, \{0,1\}, \delta, q_1, \{q_1\})$

where δ is given by

	0	1
q_1	q_2	q_2
q_2	q_3	q_3
q_3	q_1	q_1

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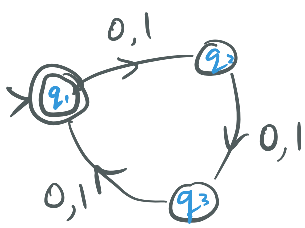
Eg:



M

$M = (\{q_1, q_2, q_3\}, \{0,1\},$
$\{(q_1, 0, q_2), (q_1, 1, q_2),$
$(q_2, 0, q_3), (q_2, 1, q_3),$
$(q_3, 0, q_1), (q_3, 1, q_1)\},$
$q_1, \{q_1\})$

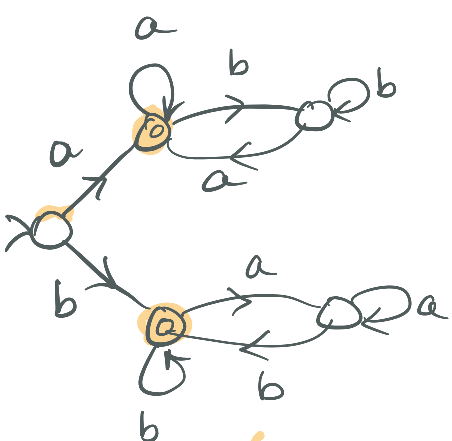
Defn: For a FA $\underline{M}$ whose alphabet is Σ

$$L(M) = \{ w \mid w \text{ is a string over } \Sigma \text{ and } M \text{ accepts } w \}.$$

We say $L(M)$ is the language recognized by (accepted by) M

Defn: FA M (over alphabet Σ) accepts a string w if w "drives" M from start state to a final state.

E.g.



M'

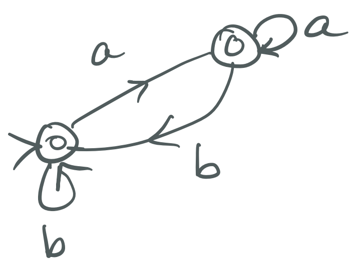
Some strings accepted by M':

a, aa, abbaba, . . .

b, bab, bb, babbab, ...

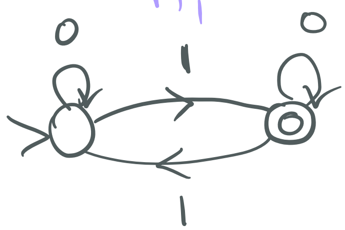
M recognizes the language of strings over $\{a,b\}$ that start and end in same letter.

Does this FA also recognize the language of strings over $\{a,b\}$ that start and end in same letter?

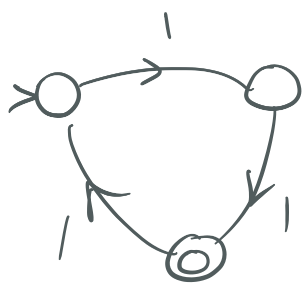


Other examples:

M_1



M_2



$\Sigma = \{0,1\}$

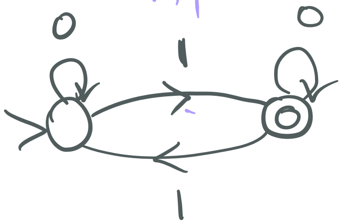
$\Sigma = \{1\}$

$L(M_1) =$

$L(M_2) =$

Other examples:

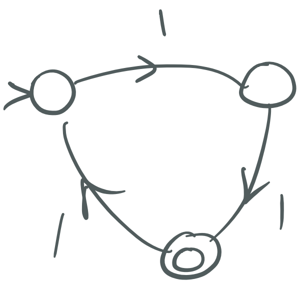
M_1



$\Sigma = \{0,1\}$

$L(M_1) = \{ \quad \}$

M_2

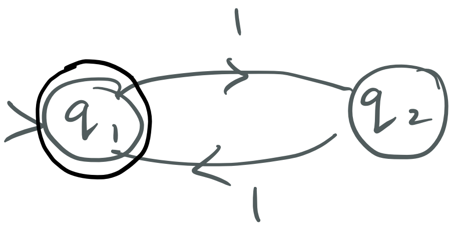


$L(M_2) = \{$

$\Sigma = \{1\}$

The "meaning" of a FA is the language it recognizes.

The "meaning" of a state can be regarded as the set of strings that can "drive" the FA from the start to that state.



M_5

"$L(q_1)$" // language of strings that can drive
// M from start to q_1

is: 1. $\varepsilon \in L(q_1)$

2. if $s \in L(q_1)$, then so is $s11$

Nothing else is in $L(q_1)$.

$L(M_5) =$ "$L(q_2)$" $= ?$

In a way, a FA language is a recursively defined object:

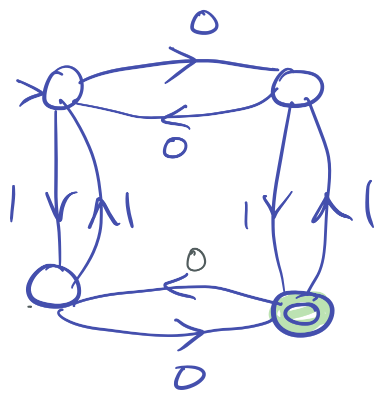
$L(M_5)$ is L_{q_2} where

L_{q_1} is 1. ε

2. $w1$ where $w \in L_{q_2}$.

L_{q_2} is 1. $w1$ where $w \in L_{q_1}$.

Claim: L_{q_2} is $\{w : w$ is an odd-length string of 1's$\}$.



What Language is recognized by this FA?

The Regular Operations on Languages

Defn: Let A and B be languages.

Define the following operations:

Union $A \cup B = \{x \mid x \in A \text{ or } x \in B\}$

Concat $A \circ B = \{xy \mid x \in A \text{ and } y \in B\}$

Star $A^* = \{x_1 x_2 \cdots x_k \mid k \geq 0 \text{ and } x_i \in A \ \forall i\}$

E.g. $A = \{ab, aa\}$ $B = \{bb\}$

$A \cup B = \{ab, aa, bb\} = \{bb, aa, ab\}$

$A \circ B = \{abbb, aabb\}$

$A^* = \{\varepsilon, aa, ab, aaaa, aaab, abaa, abab, \ldots\}$

$B^* = \{\varepsilon, bb, bbbb, \ldots\}$

note shortlex order

Defn A set A is closed under binary opn "$\blacksquare$" if $w_1 \blacksquare w_2 \in A$ whenever $w_1 \in A$ and $w_2 \in A$

Quiz: $\mathbb{N} = \{1, 2, 3, \ldots\}$

$\mathbb{W} = \{0, 1, 2, \ldots\}$

$\mathbb{Z} = \{\ldots -2, -1, 0, 1, 2, \ldots\}$

$\mathbb{W}$ closed under $+$?

$\mathbb{Z}$ closed under $-$?

$\mathbb{Z}$ closed under $\times$ (mult) ?

$\mathbb{Z}$ closed under $\div$?

W closed under $-$ (minus)

W closed under $\boxed{\text{negation}}$?

Def$^{\underline{n}}$: A set is closed under unary op "Δ" if $\Delta w \in A$ whenever $w \in A$.