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Pumping Lemma

– using it to prove languages are not regular –

For each of the following languages, either prove it is **regular** or prove it is **not regular**

$L_1 = \{ a^i b^j \mid i, j \geq 0 \text{ and } i + j = 5 \}$ bbbbb + abbbb + aabbb + aaabb + aaaab + aaaaa

$L_2 = \{ a^i b^j \mid i, j \geq 0 \text{ and } i - j = 5 \}$

$L_3 = \{ a^i b^j \mid i, j \geq 0 \text{ and } i - j \equiv 0 \bmod 5 \}$

$L_4 = \{ a^i b^j \mid 0 \leq i \leq j \leq 2000 \}$

$L_5 = \{ w \in \{a,b\}^* \mid w = w_1 w_2 \text{ where } |w_1| = |w_2| \text{ and } \#_a(w_1) = \#_a(w_2) \}$ Tricky!

$L_6 = \{ w \in \{a-z\}^* \mid$ every letter in w appears at least twice $\}$

e.g. unprosperousness

$L_7 = \{ w \in \{a,b\}^* \mid \#(w_a) \neq \#(w_b) \}$

hint: use a closure theorem ⇐ we'll cover it today in class

$L_8 = \{ a^i b^j c^k \mid k = i+j \}$ // and $i, j, k \geq 0$

PL: $\forall$ RL L $\exists$ a constant p such that $\forall w \in L$,
$|w| \geq p$, we have: $w = xyz$ such that i) $|y| > 0$,
ii) $|xy| \leq p$, and $xy^i z \in L$ $\forall$ $i = 0, 1, 2, \ldots$

aaabbccccc

Claim:

9. $\{ a^i b^j c^k \mid \text{if } i=1 \text{ then } j=k \}$ Tricky!

$L_7 = \{ w \in \{a,b\} \mid \#_a(w) \neq \#_b(w) \}$

Claim: L_7 is not regular.

Proof: BWOC. Suppose L_7 is regular.

Then so is $\overline{L_7} \cap L(a^*b^*)$

Observe that $\overline{L_7} \cap L(a^*b^*) = A^n B^n$

But $A^n B^n$ is not regular!

$\Rightarrow\Leftarrow$

$\therefore$ L_7 is not regular. ▨

$L_5 = \{ w \in \{a,b\}^* \mid w = w_1 w_2, |w_1| = |w_2|, \text{ and } \#_a(w_1) = \#_a(w_2) \}$

Claim: L_5 is not regular.

Proof: