Jan 21 2025

Equivalence of r.e. and FA.

Theorem: A language is regular iff $\exists$ a r.e. that describes it.

Proof: ($\Leftarrow$ i.e re $\to$ NFA)

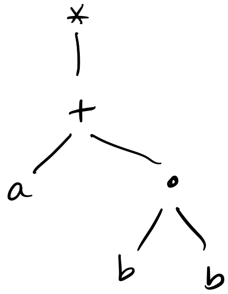
1. σ, $\sigma \in \Sigma$
2. ε
3. $\emptyset$
4. $(R_1 + R_2)$
5. $(R_1 \cdot R_2)$
6. (R_1^*)

The resulting FA clearly accepts the language described by the r.e. ∎

Aside: Let's do an example.

Using the construction:

$(a+bb)^*$

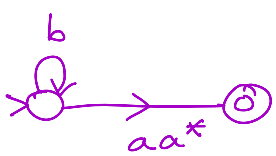


"Grok + Blurt" method:

I. ($\Rightarrow$: FA $\to$ r.e.) Construction

Goal: given a FA M, construct a r.e. R such that $L(M) = L(R)$.

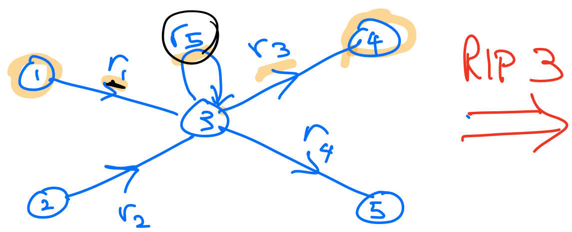
To do this, we invent something we will call a "generalized FA", where the transitions are labelled with r.e.'s.

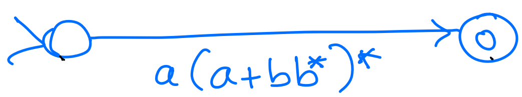


We step-by-step RIP some state and replace the affected paths with new r.e.'s.

FA-to-RE: Given a FA M

1. Ensure start state has no "in" edges.
   - add a new state if necessary... while ensuring the gen-FA accepts the same language
2. Ensure ∃ a single accept state w/ no "out" edges - introduce a new state if necessary... while ensuring etc.
3. While ∃ > 2 states:
   RIP a state that is neither start nor accept.
   ensure the new gen FA accepts the same language

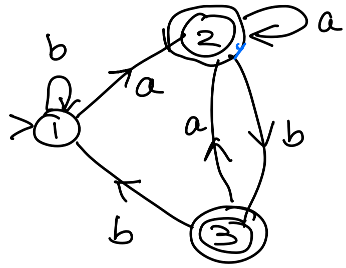




⇐ if we have just one transition, from start start to the final state, we are done. ∎

r.e. is:

Eg



Grok + Blurt:

What is the closure of $\{6, 8\}$ under <u>subtract</u>

$$\{ 6, 8, 0, 2, -2, \ldots \}$$