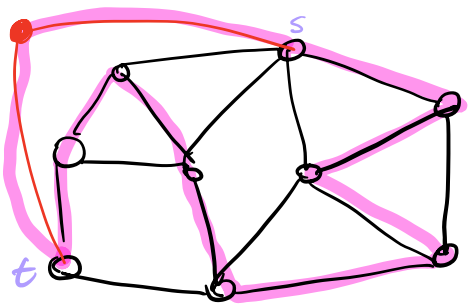
HamPath DHamPath (← directed.) HamCycle DHamCycle

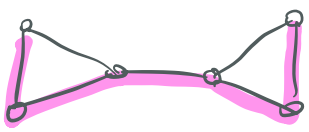
stHamPath Max Weighted Cycle

$st\text{-}HamPath \leq_p HamCycle$.



∃ a HamCycle detector but you want to know ∃ an s-t Ham path?

$HamPath \leq_p HamCycle$



Claim: $s\text{-}t\text{-}HamPath \leq_p HamCycle$.

Proof: Suppose $HC(G)$ decides in poly time whether a graph G has a Hamilton Cycle or not.
Then we can construct a poly-time alg $s\text{-}t\text{-}HP(G,s,t)$ whether graph G has an s-t HamPath as follows:

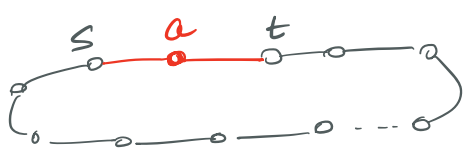
s-t-HP (graph G, vertices s, t) =

" 1. Create a graph G' from G
by adding a new vertex a to G
that is adjacent only to s and t.

2. Run HC (G').
if HC accepts G', ACCEPT.
if HC rejects G', REJECT. "

Why it works:

- if G' has a hamilton cycle, it must look like this:



and the removal of a and its two incident edges leaves an s-t Hamilton Path that also exists in G.

- it is poly-time to: copy G; add a vertex and 2 edges to the new copy; and to

execute the final "if" statement.