Recall:

A_{TM} is undecidable.

Proof: BWOC. $\nexists$ A_{TM} is decided by a TM A

Then we can construct the following TM S

S = "On input $\langle M \rangle$:

1. Run A on input $\langle M, \langle M \rangle \rangle$
   - if A accepts, ACCEPT.
   - if A rejects, REJECT."

decider for language we are trying to prove is undecidable.

decider for known undecidable

Clearly S accepts $\langle M \rangle$ iff $M \in SelfAcc$, and rejects otherwise.

$\therefore$ S decides $SelfAcc$. But $SelfAcc$ is undecidable.

$\Rightarrow\Leftarrow$

$\therefore$ $\nexists$ a TM that decides A_{TM}, and so A_{TM} is undecidable. $\blacksquare$

$Halt = \{ \langle M, w \rangle \mid M$ is a TM that halts on input $w \}$

Prove that Halt is undecidable.

Theorem: Halt is undecidable.

Proof: BWOC. $\S$ Halt is decided by TM H.

Then we can construct the following TM:

A = "on input $\langle M, w \rangle$

1. Run H on $\langle M, w \rangle$.

   if H rejects, REJECT.

2. if H accepts,

   Run M on w.

   if M accepts,

   if M rejects,

A decides A_{TM}. But A_{TM} is undecidable. $\Rightarrow\Leftarrow$

$\therefore$ H cannot exist, and Halt is undecidable.

Clearly X decides _____, which is undecidable.

undecidable language.