Assignment 5

Due April 11, 2025

1. (8 marks) Give a 3-SAT boolean expression that is satisfiable iff the following SAT expression is satisfiable. Use the construction studied in class.

$$\phi = (x \vee y \vee z \vee w \vee u \vee t) \wedge (\bar{x} \vee \bar{t})$$

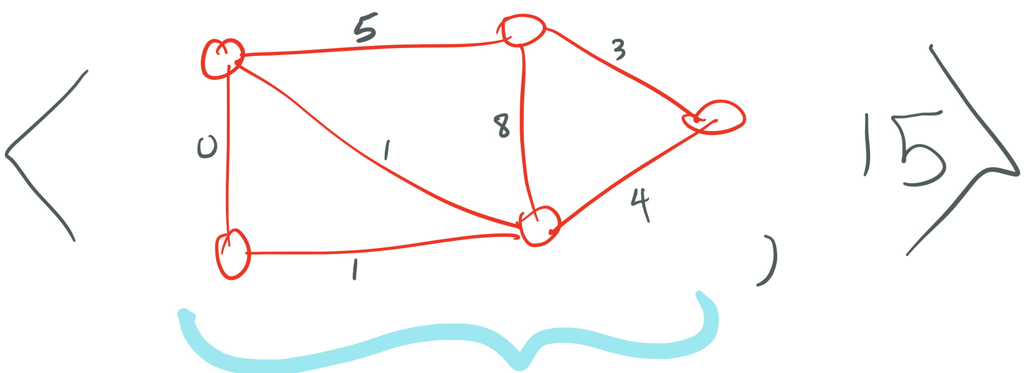
Solution:

$$\phi' = (x \vee y \vee A) \wedge (\bar{A} \vee z \vee B) \wedge (\bar{B} \vee w \vee C) \wedge$$
$$(\bar{C} \vee u \vee t) \wedge (\bar{x} \vee \bar{t} \vee \bar{t})$$

2. (8 marks) Give a poly-reduction of HamCycle to MaxWeighted Cycle:

$\text{HamCycle} = \{ \langle G \rangle \mid$ G is an undirected graph that has a Hamilton cycle $\}$

$\text{MaxWeightedCycle} = \{ \langle G, w, K \rangle \mid$ G is an undirected graph, $w: E(G) \to \mathbb{R}^+ \cup \{0\}$, $K \geq 0$, and $\exists$ a cycle in G with weight $\geq K$. (ie $\sum_{e \in cycle} w(e) \geq K$). $\}$



This is an example of a graph with a weight function on its edges.

Solution: HamCycle $\leq_p$ Max Weighted Cycle.

Let MWC be a decider for Max Weighted Cycle.

i.e $MWC(\langle G, w, k \rangle)$ returns ACCEPT iff graph G, with edge weights provided by w, has a cycle where the sum of the weights of the edges of the cycle is $\geq K$.

HC = " on input $\langle G \rangle$, where G is a graph:

1. Let w be the weight function such that $w(u,v) = 1 \;\; \forall (u,v) \in E(G)$.
2. Let $n = |V|$.

   Run $MWC(\langle G, w, n \rangle)$.

   - if MWC accepts, ACCEPT.
   - if MWC rejects, REJECT. "

Note that HC accepts G iff $\exists$ a cycle in G with n edges – i.e., if G has a Hamilton cycle.

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