Assignment 4

CSCI 320 2025

1. [8 marks]

$DiffOnString = \{ \langle M_1, M_2, w \rangle \mid M_1$ and M_2 are TMs, and exactly one of M_1, M_2 accepts $w. \}$

Is $DiffOnString$ decidable? Prove your answer.

2. [8 marks] $ALL_{TM} = \{\langle M \rangle \mid M \text{ is a TM that accepts all strings}\}$

Is ALL_{TM} decidable? Prove your answer

Hint: Have a look at the reduction we did for E_{TM} (empty).

If you are still confused about how we construct M_w, see the new notes "Making M_w"

3. WriteSymb = $\{ \langle M, w, \sigma \rangle \mid$ M is a TM which, on input $\langle w \rangle$, at some point writes the symbol σ on the tape. $\}$

[4 marks] a) Is WriteSymb recognizable? Prove your answer

[8 marks] b) Is WriteSymb decidable? Prove your answer.

Hint (b) Recall we get to "rewire" M, including adding a new tape symbol, say γ, that does not otherwise appear in M. Also we can rewire M so that certain transitions, say all transitions to a particular state, write γ on the tape.

4. BothAcceptSomeString = $\{\langle M_1, M_2\rangle \mid M_1$ and M_2 are TMs, and $\exists$ a string w that both M_1 and M_2 accept $\}$

a) [8 marks] Show that BothAcceptSomeString is recognizable.

b) [8 marks] Show that BothAcceptSomeString is not decidable.

c) [2 marks] Use a theorem from class to show that $\text{Intersection}_\emptyset$ is not recognizable:
$\text{Intersection}_\emptyset = \{\langle M_1, M_2\rangle \mid \nexists$ a string that both M_1 and M_2 accept $\}$