

Thm: A_{TM} is not decidable

Proof: BWOC. \S A_{TM} is decided by some TM, call it X .

Then we can construct a TM Y that decides

$\text{Self-Accepts} = \{ \langle M \rangle \mid M \text{ is a TM that accepts } \langle M \rangle \}$
as follows:

$Y = "$ on input $\langle M \rangle$, where M is a TM:

1. Run X on $\langle M, \langle M \rangle \rangle$
 - if X accepts, ACCEPT
 - if X rejects, REJECT. "

We note that all of Y 's instructions are doable, and halt, including the call to X , as X is a decider.
◦◦ Y halts on all inputs.

Y accepts $\langle M \rangle$ if and only if M accepts $\langle M \rangle$,
since X correctly decides A_{TM} ,
and Y rejects otherwise.

◦◦ Y correctly decides SelfAccepts .

But SelfAccepts is undecidable!

$\Rightarrow \Leftarrow$

◦◦ X cannot exist, and A_{TM} is undecidable. \square