CSCI 320 Assignment 3

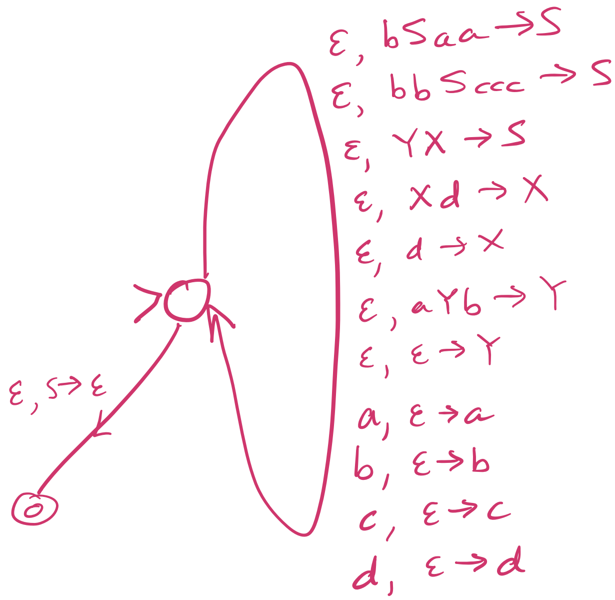
Due Tue March 18 2025

1. [4 marks] Give a bottom-up parser PDA for the following grammar:

$$S \to aaSb \mid cccSbb \mid XY$$

$$X \to dX \mid d$$

$$Y \to bYa \mid \varepsilon$$



2. [8 marks] Give a TM that recognizes the language $L_{subseq} = \{\#w\#w' \mid w \in \{a,b,c\}^*$ and w' is a (not necessarily contiguous) subsequence of $w\}$

Eg. #ababbccab#bbbca $\in L_{subseq}$

Give the TM as a transition diagram.

Use non-determinism.

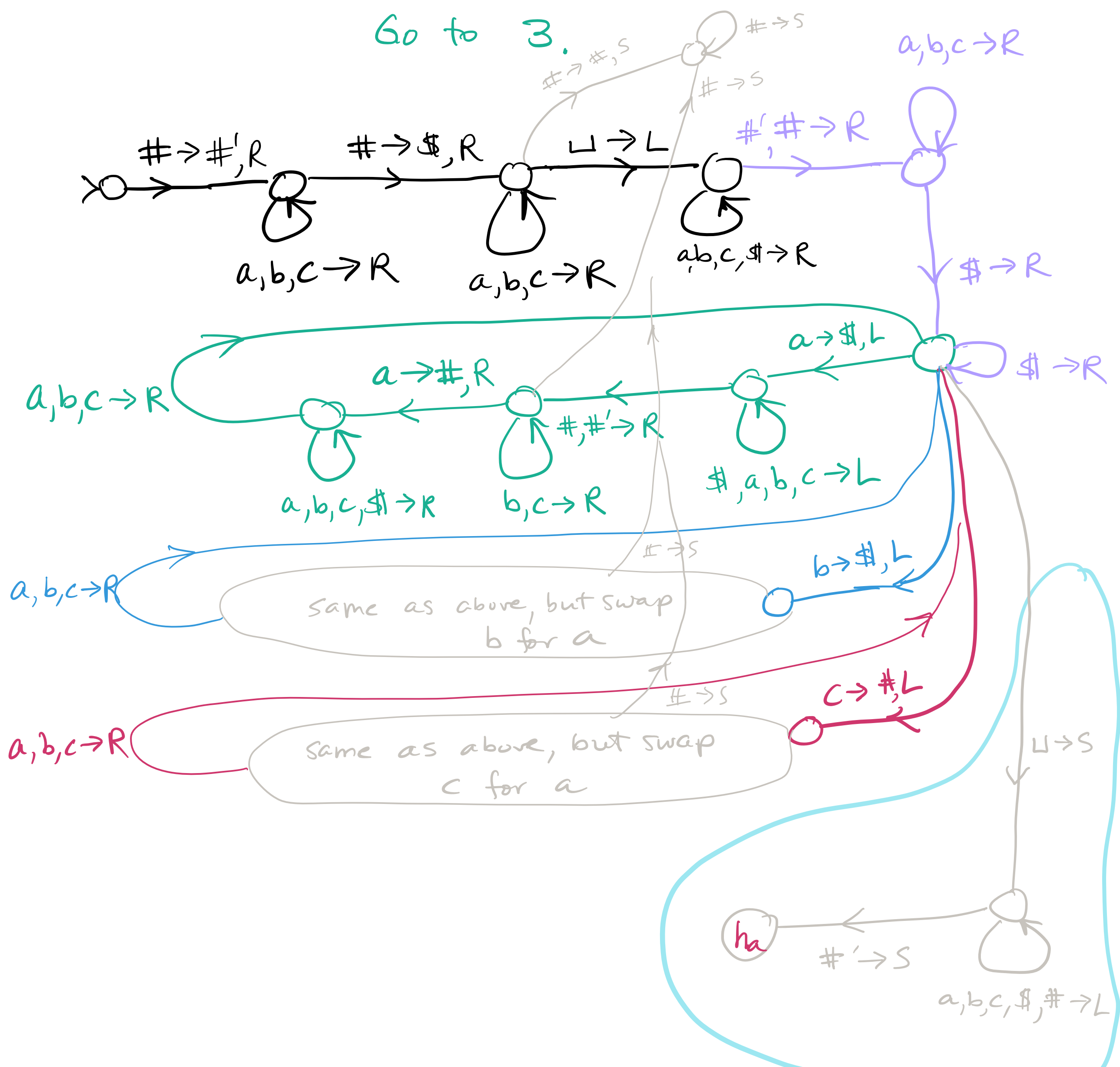
If the input is a string that is NOT in L_{subseq} make the TM loop forever.

Solution

The plan:

1. Scan R and confirm input is of format $\#(a+b+c)^*\#(a+b+c)^*$, changing 2nd # to $
2. Return L to rightmost $; move R.
3. If ⊔, go to h_a
   - 3.1 - otherwise, note the letter σ (a, b, or c) under the head
   - 3.2 - overwrite it with $
   - 3.3 - Move left to rightmost #
   - 3.4 - Move R over a's, b's, c's to first σ and overwrite with #. If there is no such σ before $, go to h_r.

3.5 – Move R over a's, b's, c's to $.
Move R while reading $.
Go to 3.



this part just moves tape head back to leftmost cell
– also I had to mark leftmost # as #' so I could recognize it.

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3. Let L be any language over $\{a,b\}$.
Show that L is countable. [4 marks]

Solution:

If L is finite, then it is countable by definition.

If L is infinite, consider the shortlex enumeration of all strings over $\{a,b\}^*$. Remove all the strings that are not in L. This leaves some ranks "blank", and the result is a loose enumeration:

1. ε
2.
3.
4. aa

⋮

Such an enumeration proves that L can be enumerated, with every string in L a finite distance down the list — ie L is countable. ▨

4. [8 marks] Prove that the ordered pairs of $\mathbb{Z} \times \{a,b\}^*$ are countable. The set is characterized as $\{(i, w) : i \in \mathbb{Z}, w \in \{a,b\}^*\}$

For example, $(5, abba)$, $(-9496, \varepsilon)$, $(0, abba)$ are in the set.

Do so by providing an enumeration scheme.
Argue that every such ordered pair will appear a finite distance down the list (i.e. will have finite rank).

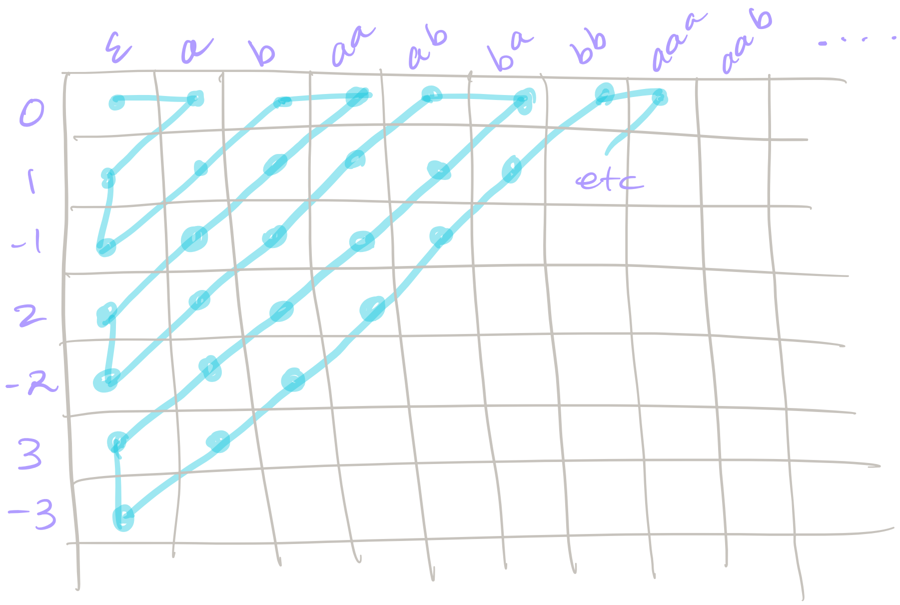
Solution:

Let us consider these two enumerations of, respectively, $\mathbb{Z}$ and $\{a,b\}^*$:

$0, 1, -1, 2, -2, 3, -3, \ldots\ldots$

$\varepsilon, a, b, aa, ab, ba, bb, aaa, \ldots\ldots$ (Shortlex)

Construct the cross-product table, and use the cross-product enumeration scheme:



sublist

each "diagonal" is finite length

$\forall$ diagonals, $\exists$ a finite # of other diagonals that come before it in the enumeration.

$\forall$ elements of $\mathbb{Z} \times \{a, b\}^*$ belong to a "diagonal" (sublist). $\blacksquare$

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5. Consider the set [8 marks]

$$ALL_{\{a,b\}} = \{ L \mid L \subseteq \{a,b\}^* \}$$

That is, $ALL_{\{a,b\}}$ is the set of all languages over $\{a,b\}$.

Show that $ALL_{\{a,b\}}$ is not countable.

Solution BWOC. $\$\ \exists$ such an enumeration

$L_1, L_2, L_3, \ldots\ldots$

Then we can create the following table of inclusion vectors for all languages over $\{a,b\}$:

	ε	a	b	aa	ab	ba	.....
L_1	0	0	1	1	0	1	...
L_2	1	0	1	1	0	0	0
L_3	0	0	0	0	0	1	0
L_4	1	1	0	0	1	1	0
L_5	1	0	1	0	1	1	1
L_6	0	0	0	0	1	1	1
$\vdots$							

Take the diagonal inclusion vector and "flip" all its bits — the result is an inclusion vector for a language that exists, but is not included in the enumeration.

$\therefore\ \nexists$ such an enumeration, and the set of languages over $\{a,b\}^*$ is not countable. $\blacksquare$