CSCI 320 Assignment 3

Due Tue March 18 2025

1. [4 marks] Give a bottom-up parser PDA for the following grammar:

$S \to aaSb \mid cccSbb \mid XY$

$X \to dX \mid d$

$Y \to bYa \mid \varepsilon$

2. [8 marks] Give a TM that recognizes the language $L_{subseq} = \{ \#w\#w' \mid w \in \{a,b,c\}^* \text{ and } w' \text{ is a (not necessarily contiguous) subsequence of } w \}$

Eg. $\#ababbccab\#bbbca \in L_{subseq}$

Give the TM as a transition diagram.

Use non-determinism.

If the input is a string that is NOT in L_{subseq} make the TM loop forever.

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3. Let L be any language over $\{a, b\}$.
Show that L is countable. [4 marks]

4. [8 marks] Prove that the ordered pairs of $\mathbb{Z} \times \{a,b\}^*$ are countable. The set is characterized as $\{(i, w) : i \in \mathbb{Z}, w \in \{a,b\}^*\}$

For example, $(5, abba)$, $(-9496, \varepsilon)$, $(0, abba)$ are in the set.

Do so by providing an enumeration scheme. Argue that every such ordered pair will appear a finite distance down the list (i.e. will have finite rank).

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5. Consider the set [8 marks]

$$ALL_{\{a,b\}} = \{ L \mid L \subseteq \{a,b\}^* \}$$

That is, $ALL_{\{a,b\}}$ is the set of all languages over $\{a,b\}$.

Show that $ALL_{\{a,b\}}$ is not countable.