

Decidability, Recognizability, Undecidability

Assignment 3. Due Mon March 23, 2026 4:00 pm.

4. $\text{RejectsSomeString}_{\text{TM}} = \{ \langle M \rangle \mid M \text{ is a TM and } M \text{ rejects some string} \}$

$\frac{1}{8}$ a) The language given above is not decidable – prove this.

$\frac{1}{8}$ b) Is this language recognizable? Prove your answer.

5. $\text{Loops}_{\text{TM}} = \{ \langle M, w \rangle \mid M \text{ is a TM that loops on input } w \}$

$\frac{1}{8}$ a) Is the language given above recognizable? Prove your answer. }

$\frac{1}{2}$ b) Is the language given above decidable? Prove your answer.

6. $\text{SixStates}_{\text{TM}} = \{ \langle M \rangle \mid M \text{ is a TM that has } \geq 6 \text{ states} \}$

$\frac{1}{8}$ Is the language above decidable? Prove your answer.

$$\text{Non}E_{\text{DFA}} = \{ \langle D \rangle \mid D \text{ is a DFA and } L(D) \neq \emptyset \}$$

Claim: $\text{Non}E_{\text{DFA}}$ is

Proof: Recall we showed that $E_{\text{DFA}} = \{ \langle D \rangle \mid D \text{ is a DFA, } L(D) = \emptyset \}$ is decidable; let X be a decider-TM for E_{DFA} .

We construct a TM Y that decides $\text{Non}E_{\text{DFA}}$ as follows:

$Y =$ " on input $\langle D \rangle$ where D is a DFA:

1. Run on input $\langle D \rangle$.
 - if X accepts, REJECT.
 - if X rejects, ACCEPT."

Y halts, because X is a decider and so always halts.

Y is correct, because $\langle D \rangle \in \text{Non}E_{\text{DFA}}$ iff $\langle D \rangle \notin E_{\text{DFA}}$.

∴ Y decides $\text{Non}E_{\text{DFA}}$. \square