

Pumping Lemma Practice

Prove the following languages are not regular by applying the pumping lemma. (And maybe closure theorems, if that makes it easier.)

1. $Bal_{(,)} = \{ w \in \{ (,) \}^* \mid w \text{ is a balanced string of parens} \}$

2. $\{ ww \mid w \in \{ a, b \}^* \}$

3. $\{ (ab)^n (ba)^n \mid n \geq 0 \}$

4. $\{ w \in \{ a, b \}^* \mid \text{either } w \text{ contains "aa" or } w = (ab)^n (ba)^n \}$

5. $\{ a^{3n} b^{2n} \mid n \geq 1 \}$ ↑
use closure.

6. $\{ w \in \{ a, b \}^* \mid \text{no suffix of } w \text{ has more } b\text{'s than } a\text{'s} \}$

And some simpler ones to warm up...

7. $\{a^n b^{2n} \mid n \geq 4\}$

8. $\{w \in \{a, b\}^* \mid w = w^k\}$

9. $\{w \in \{0, 1\}^* \mid \#_0(w) = \#_1(w)\}^*$

Theorem (Pumping Lemma)

$\forall$ Regular L , $\exists p \geq 0$ such that

$\forall w \in L$, $|w| \geq p$, $\exists x, y, z$ where

1) $xy^iz \in L \quad \forall i \geq 0$

2) $w = xyz$

3) $|xy| \leq p$

4) $|y| > 0$

1. $Bal = \{w \in \{(,)\}^* \mid w \text{ is a balanced string of parens}\}$

Claim: Bal is not regular. $w = \underbrace{(((\dots()))))}_{p} \dots \underbrace{)}_{p}$

Proof: $\nexists$ Bal is Regular, and has
pumping constant $p. = 10$

Consider the string $\underline{(}^p)^p$ (call it w)

Then by P.L. $w = xyz$, $|xy| \leq p$ so

$y = ({}^t$; and $|y| > 0$ so $t > 0$.

Then $xy^0z \in L$ ie $\underline{({}^{p-t})^p} \in L$.

$\Rightarrow \Leftarrow$

$\uparrow$
not in language.

$$L_3 = \{ (ab)^n (ba)^n \mid n \geq 0 \}.$$

ab ab abba baba

Claim: L_3 is not regular.

Proof: $\S$ L_3 is regular, and has

pumping constant p . $\underbrace{abab \dots ab}_p \underbrace{baba \dots ba}_p$

Consider the string $\underline{(ab)^p(ba)^p} = w$.

Then $w = xyz$ where $|x| \leq p$, $|y| > 0$.

So:

Case 1. $y = b(ab)^r$ or $(ab)^r a$

then y does not have $\#_a(y) = \#_b(y)$

So $\#_a(xz) \neq \#_b(xz)$

but $xz \in L$ (by pumping down)

$\Rightarrow$ (all strings s in L have $\#_a = \#_b$)

$ababab \dots abababab \dots ba$
 $\underbrace{\hspace{10em}}_{p \text{ times}}$

Case 2: $y = (ab)^r$

Then pumping down once yields

$(ab)^{p-r} (ba)^p$, which does not have bb in middle since $r \neq 0$.

$\Rightarrow \Leftarrow$ (all strings in L have bb in the middle.)

Claim: $L_7 = \{ w \in \{0,1\}^* \mid \#_0(w) = \#_1(w) \}$.

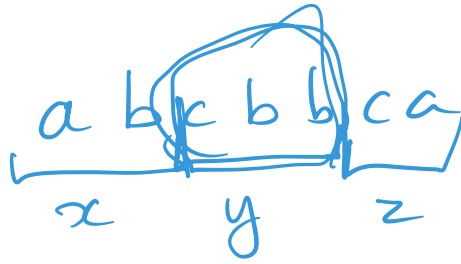
Proof: Let P be L_7 's pumping const.

Consider $0^P 1^P$.

Then $y = 0^t$ for some $t > 0$.

then $\boxed{xy^2z} = 0^{P+t} 1^P$,

which is not in L_7 . $\square$ QED



Then $xy^0z = abca$

$xy^1z = abcbbca$

$xy^2z = abcbcbcbca$