Pumping Lemma Practice

Prove the following languages are <u>not</u> regular by applying the pumping lemma. (And maybe closure theorems, if that makes it easier.)

1. $Bal_{(,)} = \{ w \in \{(,)\}^* \mid w \text{ is a balanced string of parens} \}$

2. $\{ ww \mid w \in \{a,b\}^* \}$

3. $\{ (ab)^n (ba)^n \mid n \geq 0 \}$

4. $\{ w \in \{a,b\}^* \mid \text{either } w \text{ contains "aa" or } w = (ab)^n (ba)^n \}$

   ↑ use closure.

5. $\{ a^{3n} b^{2n} \mid n \geq 1 \}$

6. $\{ w \in \{a,b\}^* \mid \text{no suffix of } w \text{ has more } b\text{'s than } a\text{'s} \}$

And some simpler ones to warm up...

7. $\{ a^n b^{2n} \mid n \geq 4 \}$

8. $\{ w \in \{a,b\}^* \mid w = w^R \}$

9. $\{ w \in \{0,1\}^* \mid \#_0(w) = \#_1(w) \}$ *

Theorem (Pumping Lemma)

$\forall$ Regular L, $\exists\ p \geq 0$ such that

$\forall w \in L$, $|w| \geq p$, $\exists\ x, y, z$ where

1) $xy^iz \in L \ \forall\ i \geq 0$

2) $w = xyz$

3) $|xy| \leq p$

4) $|y| > 0$

1. $Bal = \{ w \in \{(,)\}^* \mid w$ is a balanced string of parens $\}$

Claim: Bal is not regular. $w = \underbrace{((((\ldots(}_{p}\underbrace{))))\ldots)}_{p}$

Proof: $\exists$ Bal is Regular, and has pumping constant $p. = 10$

Consider the string $(^p)^p$ (call it w)

Then by P.L. $w = xyz$, $|xy| \leq p$ so $y = (^t$; and $|y| > 0$ so $t > 0$.

Then $xy^0z \in L$ ie $\underbrace{(^{p-t})^p}_{}$ $\in L$.

$\Rightarrow\Leftarrow$

not in language.

$L_3 = \{(ab)^n(ba)^n \mid n \geq 0\}$.

ababab abbababa

Claim: L_3 is not regular.

Proof: Suppose L_3 is regular, and has pumping constant p. $\underbrace{abab\ldots ab}_{p}\underbrace{baba\ldots ba}_{p}$

Consider the string $(ab)^p(ba)^p = w$.

Then $w = xyz$ where $|xy| \leq p$, $|y| > 0$.

so:

Case 1. $y = a(ba)^r$ or $b(ab)^r$ or $(ab)^r a$

then y does not have $\#_a(y) = \#_b(y)$

so $\#_a(xz) \neq \#_b(xz)$

but $xz \in L$ (by pumping down)

$\Rightarrow\Leftarrow$ (all strings in L have $\#_a = \#_b$)

$\underbrace{ababab\ldots ab}_{p}\underbrace{baba\ldots ba}_{p \text{ times}}$

Case 2: $y = (ab)^r$

Then pumping down once yields $(ab)^{p-r}(ba)^p$, which does not have bb in middle since $r \neq 0$.

$\Rightarrow\Leftarrow$ (all strings in L have bb in the middle.)

Claim: $L_7 \{ w \in \{0,1\}^* \mid \#_0(w) = \#_1(w) \}$.

Proof: Let P be L_7's pumping const.

Consider $0^p 1^p$.

Then $y = 0^t$ for some $t > 0$.

then $xy^2z = 0^{p+t} 1^p$, which is not in L_7. ▨ QED

$$\underbrace{a\,b}_{x}\,\underbrace{c\,b\,b}_{y}\,\underbrace{c\,a}_{z}$$

Then $xy^0z = abca$

$xy^1z = abcbbca$

$xy^2z = ab\,cbb\,cbb\,ca$