Chapter 2 Context-Free Languages (CFL)

Human languages (speech) obey rules of a grammar.

noun verb preposition

noun phrases verb phrases prepositional phrases

- ways they can be put together into grammatical sentences are governed by a grammar.

You can use a grammar to generate strings in a language, or to check a string to determine if it is grammatical.

<sentence> → <noun-phrase> <verb-phrase>

<noun-phrase> → <cmplx-noun> | <cmplx-noun> <prep-phrase>

<verb-phrase> → <cmplx-verb> | <cmplx-verb> <prep-phrase>

<cmplx-noun> → <article> <noun>

<cmplx-verb> → <verb> | <verb> <noun-phrase>

<article> → a | the | her

<noun> → cat | dog | paw

<verb> → swats | likes | sees

<prep> → with

<prep-phrase> → <prep> <noun-phrase>

<sentence> ⇒ <noun p> <verb p>

⇒ <cmplex n> <verb p>

⇒ <art> <noun> <verb phrase>

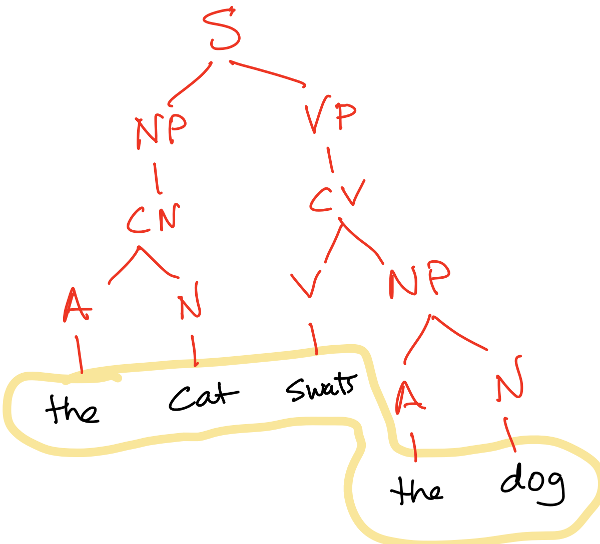
⇒ the <noun> <verb phrase>

$\Rightarrow$ the cat <verb phrase>

$\Rightarrow$ the cat

$\vdots$

$\rightarrow$ the cat swats the dog.



derived string is on the leaves L – R + consist of terminals.

Definition 2.2 A Context-Free Grammar (CFG) is a 4-tuple (V, Σ, R, S) where

1. V is a finite set of variables
2. Σ is a finite set of terminals $\Sigma \cap V = \emptyset$.
3. R is a finite set of rules, each being a variable and a string from $(\Sigma \cup V)^*$
4. $S \in V$ is start variable.

If u, v, w are strings $\in (\Sigma \cup V)^*$ and
$A \to w$ is a rule in the grammar, then we say

uAv "yields (in one step)" uwv

$$uAv \Rightarrow uwv$$

We say "u derives v" write $u \overset{*}{\Rightarrow} v$

if : $u = v$, or

$\exists \quad u \Rightarrow u_1 \Rightarrow u_2 \Rightarrow \dots \Rightarrow v$

The <u>language</u> $L(G)$ of grammar G is
$$\{ w \in \Sigma^* \mid S \overset{*}{\Rightarrow} w \}$$

Eg $G = (\{S\}, \{a, b\}, R, S)$

"or"

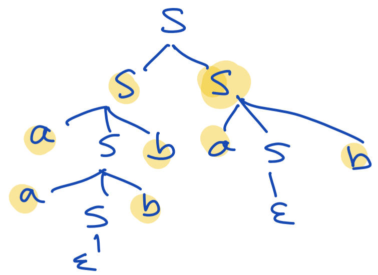
and R is $S \to aSb \mid SS \mid \varepsilon$

(that is the same as
$$S \to aSb$$
$$S \to SS$$
$$S \to \varepsilon$$
)

· aabb

$S \Rightarrow$ $\qquad\qquad \Rightarrow aabb$ $\qquad$ S

aabbab ?



aabbab

*Leftmost derivation

$S \Rightarrow SS \Rightarrow aSbS \Rightarrow aaSbbS \Rightarrow aabbS$

$\Rightarrow aabbaSb \Rightarrow aabbab.$

– always apply production rule to the leftmost variable.

Rightmost derivation:

$S \Rightarrow SS \Rightarrow SaSb \Rightarrow Sab \Rightarrow aSbab \Rightarrow aaSbbab$

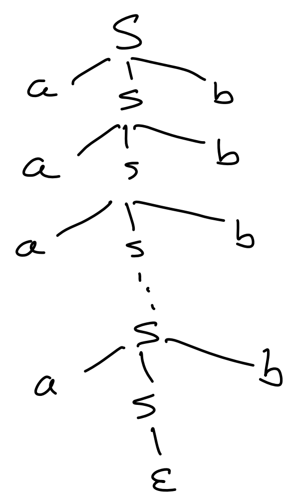
$\Rightarrow aabbab$

Fun with Grammars.

$A^nB^n = \{a^n b^n \mid n \geq 0\}.$

G: $S \rightarrow aSb \mid \varepsilon$

$\{\varepsilon, ab, aabb, \ldots, \}.$



A string w is $\in L(G)$ iff.

1. $w = \varepsilon$
2. $w = aw'b$ where $w' \in L(G)$.

Bal is context-Free:

$S \to SS \mid (S) \mid \varepsilon$

Bal: 1. $\#_(= \#_)$
2. In all prefixes $\#_(\geq \#_)$

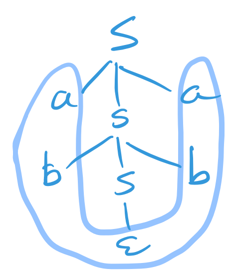
To argue that a CFG G is correct for a language X, (i.e. that $L(G) = X$), we need to show:

1. $w \in L(G) \Rightarrow w \in X$
2. $w \in X \Rightarrow w \in L(G)$

1. $\{w \in \{a,b\}^* \mid \#_a(w) = \#_b(w)\}$.
2. $EvenPal = \{w \in \{a,b\}^* \mid |w| \text{ is even}, w \text{ is palindrome}\}$

$EvenPal = \{ww^R \mid w \in \{a,b\}^*\}$

$S \to aSa \mid bSb \mid \varepsilon$



a b b a

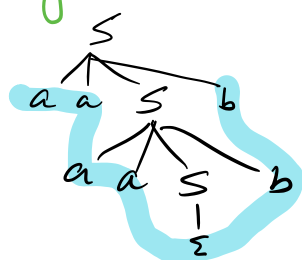
$\text{OddPal} = \{ w \in \{a,b\}^* \mid |w| \text{ is odd, and } w = w^R \}$

$S \to aSa \mid bSb \mid a \mid b$

Pal: $S \to aSa \mid bSb \mid a \mid b \mid \varepsilon$

$L_1 = \{ a^{2k} b^k : k \in \{0, 1, 2, \ldots\} \}$

$L_2 = \{ a^i b^j : i \text{ is odd, } j \text{ is even} \}.$



L_1: $S \to aaSb \mid \varepsilon$

L_2: $(aa)^* a (bb)^*$

$S \to AB$
$A \to aAa \mid a$
$B \to bBb \mid \varepsilon$

$S \to AB$
$A \to aaA \mid a$
$B \to Bbb \mid \varepsilon$

$S \to AaB$
$A \to aaA \mid \varepsilon$
$B \to bbB \mid \varepsilon$

$S \to aaS \mid Sbb \mid a$