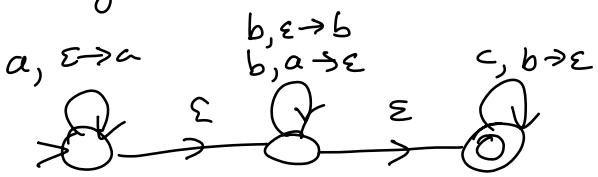
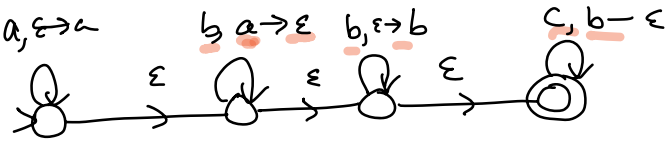
Recall from last time $\{ a^i b^{i+j} c^j \mid i, j \geq 0 \}$. ← Find a CFG.

Is that language inherently non-deterministic?

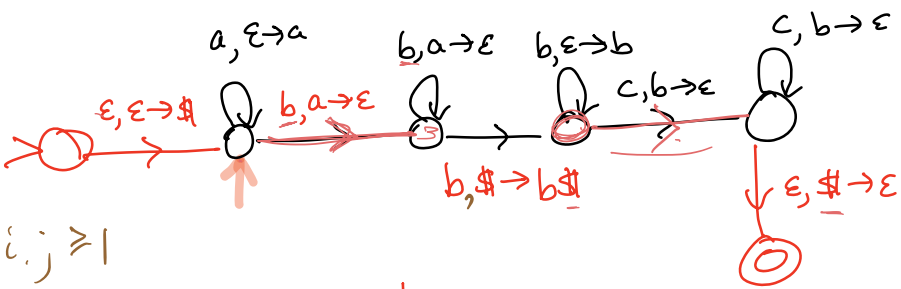



⇓ more deterministic.




Note Your text always use bottom-of-stack marker.
Sipser's def$^n$ of PDA accepting a string is a little different but equivalent.

⇓ more deterministic: detect bottom-of-stack.



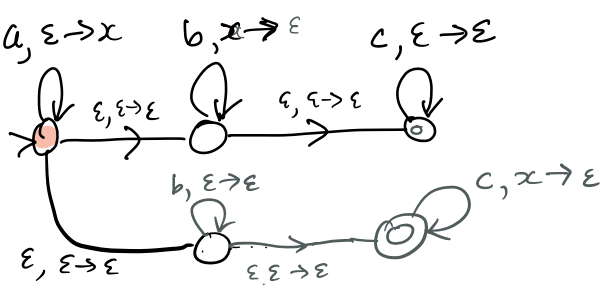

$L = \{ a^i b^{i+j} c^j \mid i, j \geq 1 \}$

deterministic

Can we always remove the non-determinism?

$\{ a^i b^j c^k \mid i = j \text{ or } i = k \}$.

This PDA has no deterministic equivalent.



We may also want to detect end-of-string
end-of-input.

Then we say, instead of finding a PDA that recognizes $L$, we find a PDA for $L\#$.

↑ "end of string" symbol at end of each string in $L\#$

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**Theorem 2.20**

A language is CF iff ∃ a PDA that recognizes it.

ie ∃ a CFG that generates it

ie ∃ a CFG in CNF that generates it

**Proof I** ($\Rightarrow$) $\forall$ $L$ is CF ie has a CFG $G_L$.

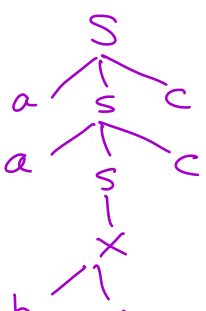
[want to show: ∃ a PDA that recognizes $L$]

We will construct a PDA that simulates deriving a string in the grammar $G_L$.

Eg $S \to aSc \mid X$
$X \to bX \mid \varepsilon$

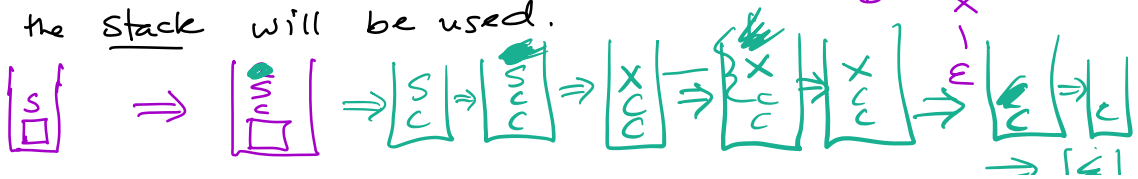
$\{a^i b^j c^i \mid i, j \geq 0\}$

aabcc



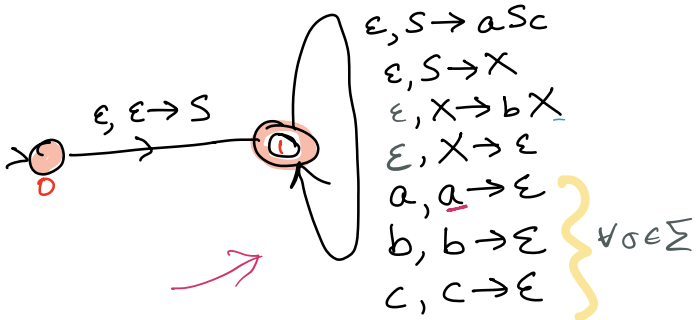

$S \Rightarrow aSc \Rightarrow aaScc \Rightarrow aaXcc \Rightarrow aabXcc$
$\Rightarrow aabcc$

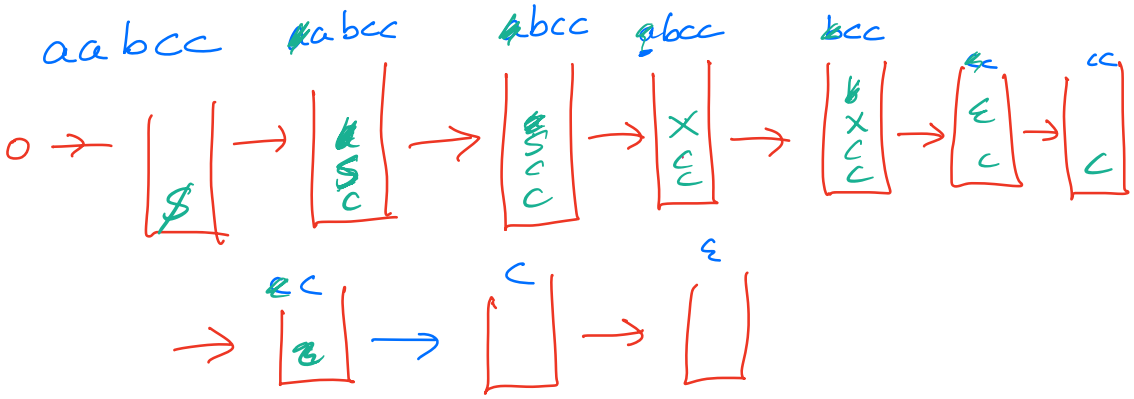
How the stack will be used.




Top-Down Parser for $S \to aSc \mid X$, $X \to bX \mid \varepsilon$.

aabcc




In general

$\forall$ CFG $G = (V, \Sigma, R, S)$ $\exists$ a Top-Down Parser PDA $M$ that recognizes $L(G)$ where

$$M = (\{q_0, f\}, \Sigma, \Sigma \cup V, \delta, q_0, \{f\})$$

and where the transitions of $\delta$ are as follows:

$$\delta(q_0, \varepsilon, \varepsilon) = \{(f, S)\}$$

$$\forall \sigma \in \Sigma, \quad \delta(f, \sigma, \sigma) = \{(f, \varepsilon)\}.$$

$$\forall Z \in V, \quad \delta(f, \varepsilon, Z) = \bigcup_{Z \to \gamma} \{(f, \gamma)\}$$

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rules in R.

*

## II ($\Leftarrow$) $\forall$ PDA $M$, $\exists$ CFG $G$ s.t. $M$ recognizes $L(G)$

First, simplify $M$ so that:

1. it has a single accept state.
2. each transition EITHER — pushes a symbol / Not both.
   — pops a symbol

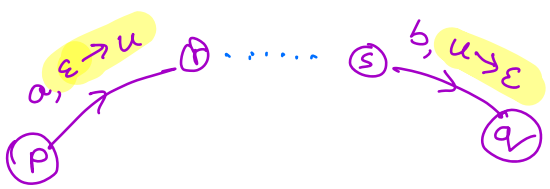

What we are interested in doing is creating variables

$A_{pq}$ — has the property that $\exists$ a derivation $A_{pq} \Rightarrow \cdots \Rightarrow$ abbccb ← an example string of $\Sigma^*$

iff $\exists$ a way $M$ can go from state $p$ to state $q$ consuming input abbccb

net result on stack is $\varepsilon$

$\forall$ pairs of states $p, q$




then $A_{pq} \rightarrow a A_{rs} b$

Remember:
$A_{pq}$ "means"
"strings that can take M from p to q."

$$A_{pq} \rightarrow A_{pr} A_{rq}$$

$$A_{pp} \rightarrow \varepsilon$$

Start symbol is : $A_{q_0 q_{accept}}$