Last slide on P.L.

$Prime_a = \{ a^n \mid n \text{ is prime} \}$ 2, 3, 5, 7, 11, ...

Claim: $Prime_a$ is not regular.

Proof: BWOC. Suppose $Prime_a$ is regular, and let k be its pump const.

Let t be the smallest prime $\geq k+2$.

Consider the string $w = a^t$. Note $w \in Prime_a$ and $|w| \geq k$.

(in fact, $|w| \geq k+2$).

$$w = \underbrace{a\ a\ \overbrace{a\ a\ a}^{y} \ldots a}_{k}\ \underbrace{a\ a}_{\geq 2 \text{ extra } a\text{'s}}$$

By PL, $w = xyz$ where $|y| > 0$, $|xy| \leq k$, and $xy^iz \in Prime_a$ $\forall i \geq 0$.

i.e. $a^{|x|}\, a^{i \cdot |y|}\, a^{|z|} \in Prime_a$. $\forall i \geq 0$. ($|z| \geq 2$.)

in particular when $i = |x| + |z|$, the result of pumping up is

$$a^{|x|}\, a^{(|x|+|z|)\,|y|} \cdot a^{|z|}$$

$$= a^{\underbrace{(|x|+|z|)}_{\geq 2}\underbrace{(|y|+1)}_{\geq 2}}$$ ... but $(|x|+|z|)(|y|+1)$ is not prime!

$\Rightarrow\Leftarrow$ $\therefore$ $Prime_a$ is not regular. ▨

Closure Theorems

Recall that the class RL is closed under

- union
- concatenation
- $*$
- complement

$\Longrightarrow$ intersection

- letter substitution

$a \to 0110$
$b \to 1001$

"every a is followed immed by b"
"every 0 is followed immed by 1"

We can use these facts to help find nicer proofs of Non regularity.

$$L = \{ a^i b^j \mid i,j \geq 0 \text{ and if } i = 1 \text{ then } j \text{ is prime} \}$$

Claim: L is not regular

Proof: BWOC, $\S$ L is regular. Let p denote L's pumping constant. Let q be smallest composite $q \geq p+2$
Consider the string $a^p b^q$?

Proof: If L is regular, then so is $L \cap L(ab^*) = L'$

Known reg language

Note that $L' = \{ab^i \mid i \text{ is prime}\}$.

Claim: L' is not regular.

Proof: BWOC. $\exists$ L is reg and has pumping const p.

Let r be the smallest prime $r \geq p+2$.

Consider ab^r, which is in L' and is long enough so

$ab^r = xyz$ where 1. $xy^iz \in L'$, 2. $|y|>0$ 3. $|xy| \leq p$.

So:

i) $y = ab^t$ for some $t \geq 0$. But then pumping down gets rid of all a's $\Rightarrow\Leftarrow$ (see L' defn).

ii) $y = b^t$ for some $t \geq 1$.

⋮

use the Prime$_a$ proof.

L' is not regular. $\square$

Since L' is not regular, neither is L. $\blacksquare$

Recalling from earlier slides that

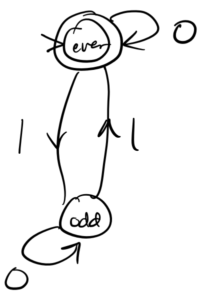
$$L_1 \in RL \text{ and } L_2 \in RL \Rightarrow L_1 \cup L_2 \in RL$$

<u>Proof</u> that if L_1 and L_2 are regular, so is $L_1 \cup L_2$

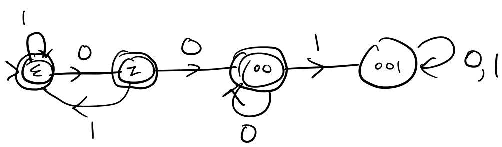
"Proof by example"

$\Sigma = \{0,1\}$

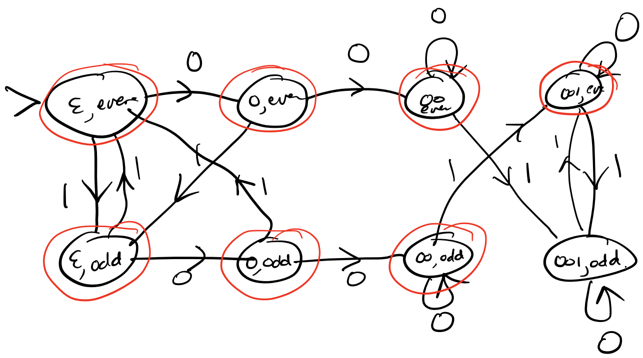
"$\#_1(w)$ is even"



"has no 001 substring"



Union



Intersection