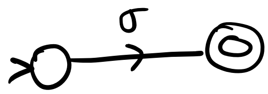
Jan 21 2025

Equivalence of r.e. and FA.

Theorem: A language is regular iff $\exists$ a r.e. that describes it.

Proof: ($\Leftarrow$ i.e re $\to$ NFA)

1. σ, $\sigma \in \Sigma$

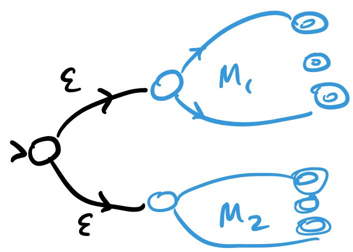


2. ε

3. $\emptyset$

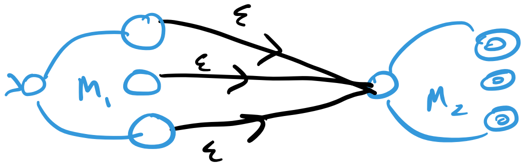
4. $(R_1 + R_2)$

$L(M_1) = L(R_1)$

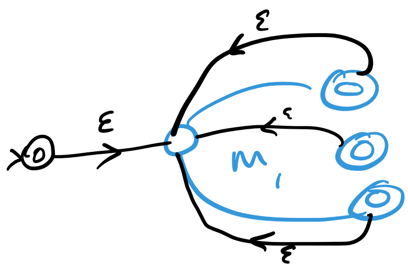


$L(M_2) = L(R_2)$

5. $(R_1 \cdot R_2)$



6. (R_1^*)

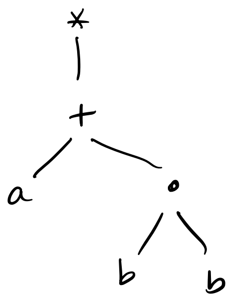


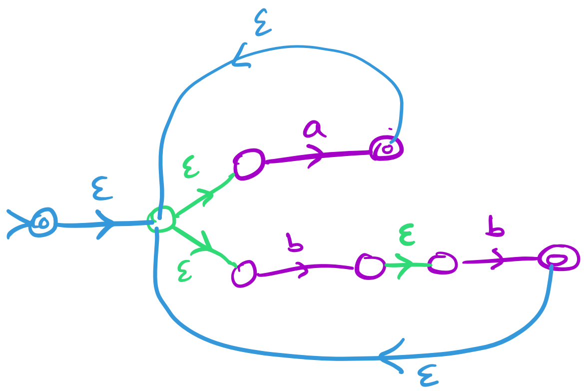
The resulting FA clearly accepts the language described by the r.e. $\blacksquare$

Aside: Let's do an example

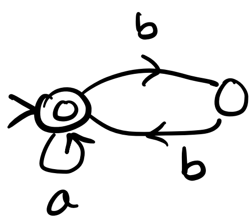
Using the construction:

$(a+bb)^*$





"Grok + Blurt" method:

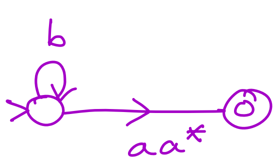


can be a NFA without ε-transitions

II. ($\Rightarrow$: FA $\to$ r.e.) Construction

Goal: given a FA M, construct a r.e. R such that $L(M) = L(R)$.

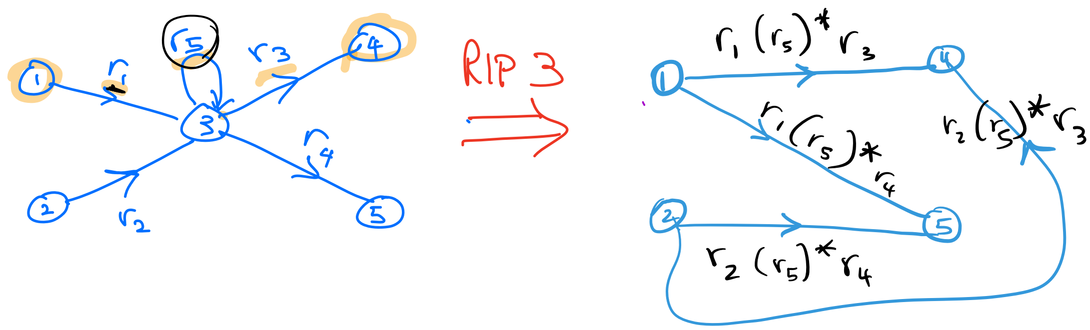
To do this, we invent something we will call a "generalized FA", where the transitions are labelled with r.e.'s.

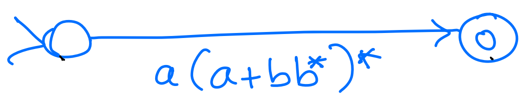


We step-by-step RIP some State and replace the affected paths with new r.e.'s.

FA-to-RE: Given a FA M

1. Ensure start state has no "in" edges.
   - add a new state if necessary... while ensuring the gen-FA accepts the same language
2. Ensure ∃ a single accept state w/ no "out" edges - introduce a new state if necessary.... while ensuring etc.
3. While ∃ > 2 states:
   RIP a state that is neither start nor accept.



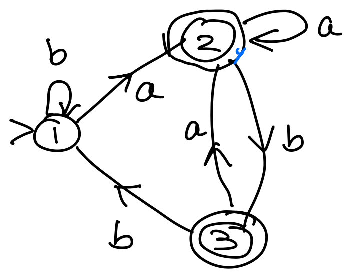


⇐ if we have just one transition, from start start to the final state, we are done.

r.e. is: $a(a+bb^*)^*$

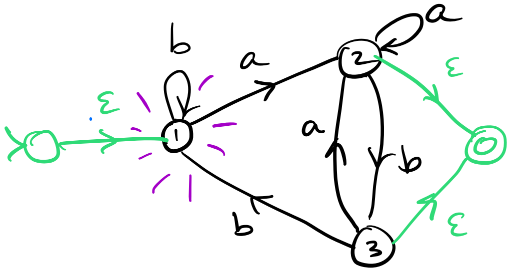
∎

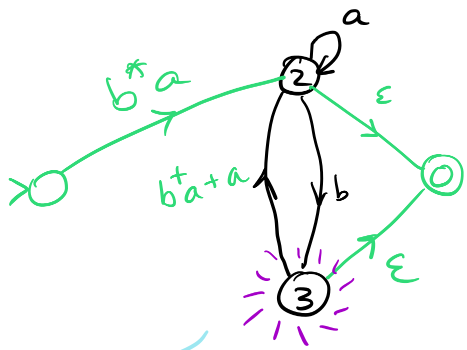
Eg

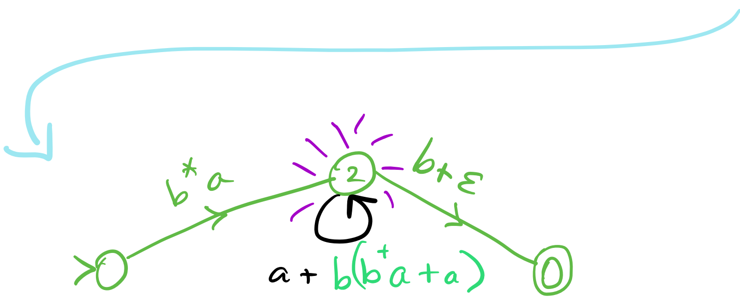


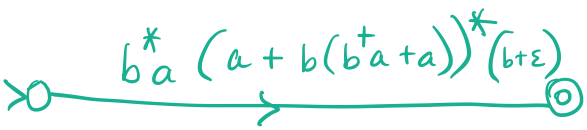
Grok + Blurt:

$$(b^*a)^+(b+\varepsilon)$$



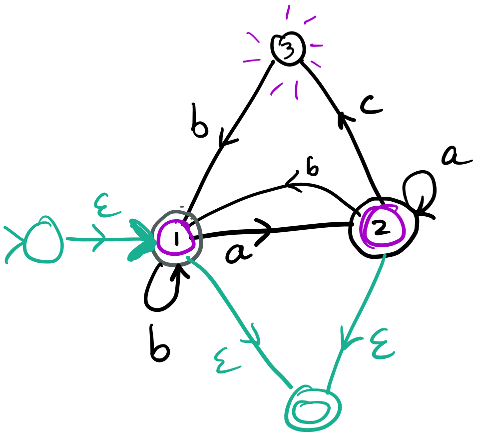


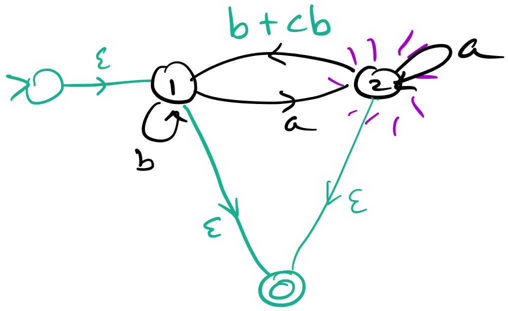


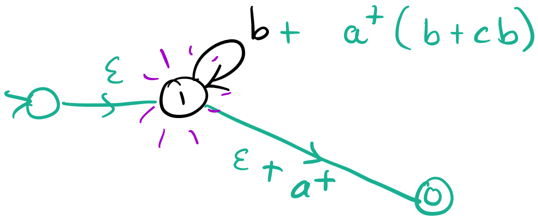


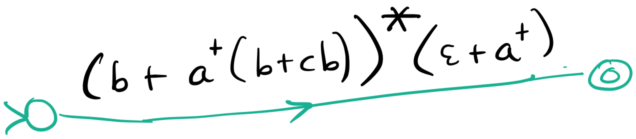
r.e. is $b^*a\big(a+b(b^+a+a)\big)^*(b+\varepsilon)$ $\blacksquare$

$(a + b + acb)^*$









re is $(b + a^+(b+cb))^* a^*$