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Definition of a Computation for Finite Automata

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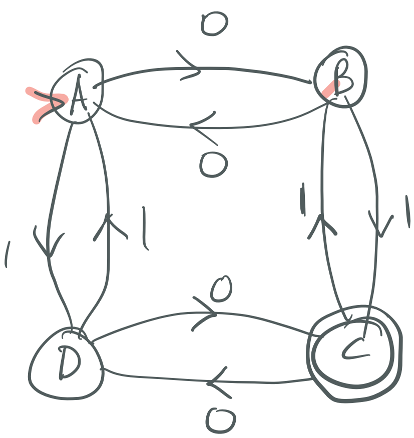


M

What is M's computation on input 011001?

$(A, 011001)$

goes to

$(B, 11001)$

$(A, 011001) \vdash (B, 11001) \vdash (C, 1001) \vdash$

$(B, 001) \vdash (A, 01) \vdash$

$(B, 1) \vdash (C, \varepsilon)$

Start State

input string

"turnstile" means "yields in one step"

Def$^{\underline{n}}$: An accepting computation is one that consumes all the input and ends at a state $f \in \underbrace{F}_{\text{the set of accepting states}}$

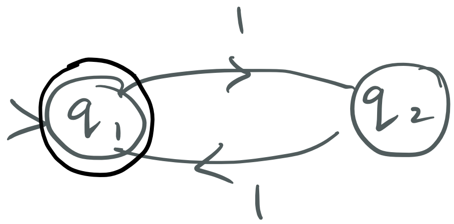
In other words (and more formally), a string w is accepted by a FA $M = (Q, \Sigma, \delta, q_0, F)$ if there is a computation that starts at

(q_0, w) and ends at (f, ε), $f \in F$.

i.e. $(q_0, w) \vdash \ldots \vdash \ldots \vdash (f, \varepsilon)$ where $f \in F$.

i.e. $(q_0, w) \vdash^{*} (f, \varepsilon)$, where $f \in F$.

"yields in 0 or more steps"



M_5.

Claim: For any string w of 1's run on M_5

$(q_1, w) \vdash^* (q_1, \varepsilon)$ if $|w|$ is even

and $(q_1, w) \vdash^* (q_2, \varepsilon)$ if $|w|$ is odd.

Proof: By induction on $|w|$

0 is even

Base: If $|w| = 0$ then $w = \varepsilon$ and

$(q_1, \cancel{\varepsilon}^{w}) \vdash^* (q_1, \varepsilon)$. i.e. claim holds.

Ind Hyp: Suppose claim holds for

$w = 1^{n-1}, \quad n > 0$.

Induction Step: Let $w = 1^n$

if n is odd, then $n-1$ is even

so $(q_1, 1^{n-1}) \vdash^* (q_1, \varepsilon)$ by Ind Hyp hence

$$(q_1, 1^{n-1} \cdot 1) \vdash^* (q_1, \varepsilon \cdot 1)$$

$$\vdash (q_2, \varepsilon).$$

So 1^n is NOT ACCEPTED if n is odd.

On the other hand, suppose n is even. Then $n-1$ is odd, and

$$(q_1, 1^{n-1}) \vdash^* (q_2, \varepsilon) \quad \text{by } \underline{\text{Ind Hyp}}$$

hence

$$(q_1, 1^n) \vdash^* (q_2, \varepsilon \cdot 1)$$

$$\vdash (q_1, \varepsilon)$$

$\therefore$ if n is even, 1^n is ACCEPTED. ∎

Restated: $L(M_5) = \{ \underbrace{1^n}_{\substack{111\ldots1 \\ n \text{ times}}} : n \text{ is even} \}.$