5. "Closure" and "Closed"

$$\mathbb{N} = \{1, 2, 3, \ldots\}$$

$$W = \{0, 1, 2, \ldots\}$$

$$\mathbb{Z} = \{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}$$

$$\mathbb{Q}^+ = \{\tfrac{a}{b} : a, b \in \mathbb{N}\}$$

$$\mathbb{Q} = \{\tfrac{a}{b} : a \in \mathbb{Z},\ b \neq 0 \text{ and } b \in \mathbb{Z}\}.$$

Def$\underline{\text{n}}$: A set S is "closed under" an operation $\blacksquare$ if

$$\forall\, a, b \in S, \quad a \blacksquare b \in S$$

(supposing $\blacksquare$ to be a <u>binary</u> operation)

Eg: Is $\mathbb{N}$ closed under $+$?
Is $\mathbb{N}$ closed under $-$?

Is $\mathbb{W}$ closed $+$? $-$?

Is $\mathbb{Q}^+$ closed under "recip" function

$\text{recip}(x) = \frac{1}{x}$?

(We can extend "closed under" to unary ops)

Is $\mathbb{Q}$ closed under negation?

Is $\mathbb{Z}$ closed under negation?

Defn: For a set S, the "closure of under $\blacksquare$" is the set

$$S \cup \{ x \blacksquare y : x, y \in S \}$$

Eg the closure of $\mathbb{N}$ under "$+$" is $\mathbb{N}$

the closure of $\mathbb{N}$ under "$-$" is $\mathbb{Z}$

the closure of $\mathbb{N}$ under "$\div$" is $\mathbb{Q}^+$

Consider an alphabet $\Sigma = \{a, b, c\}$.

What is the closure of Σ under concat "$\circ$"?

$= \{ \omega \in \Sigma^* \mid |\omega| \geq 1 \}$.

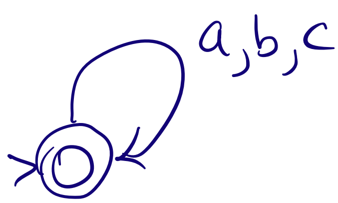
= "all nonempty strings over $\{a,b,c\}$"

$\Sigma^* = \{\varepsilon\} \cup$ closure of Σ under $\circ$.

Kleene star

$\Sigma = \{a, b, c\}$

FA for Σ^*



Closure of Σ under $\circ$