3. **Chapter 1** **Regular Languages**

From Chapter 0:

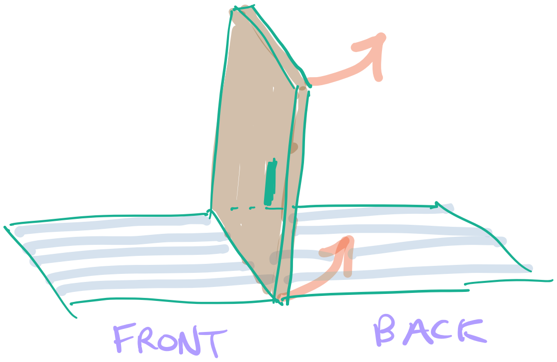
$\Sigma = \{a, b\}$ Alphabet

abba string

$\{a, aa, aaa, aba, aaaa,$ Language
$aaba, abaa, abba, aaaaa,$
$\dots\}$

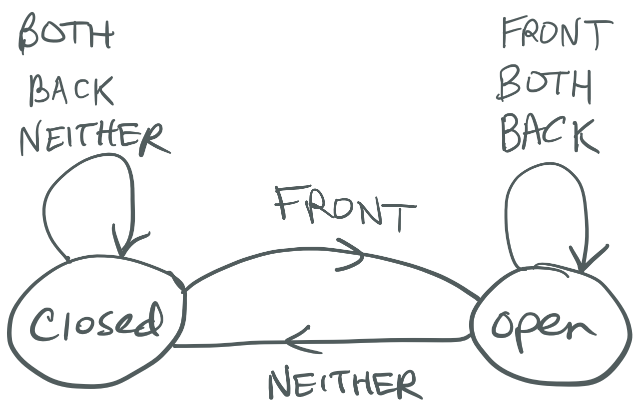
Def$^{\underline{n}}$: A language over alphabet Σ is a set of strings over Σ.

Finite Automata FA



What behavior do we want from an automatic door?

- if FRONT is occupied ("FRONT")
  then swing open ...
  unless BACK is also occupied.
  ... and assuming door is closed.
  ⋮



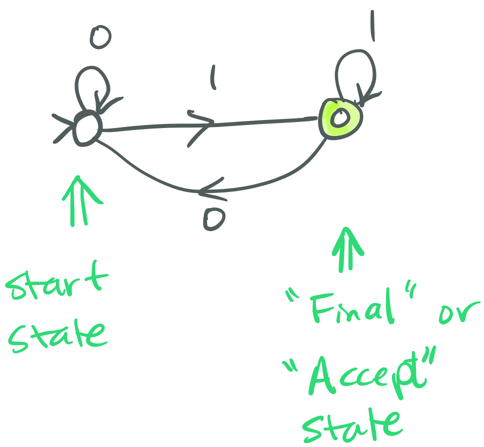
Transition Table for automatic door controller:

input from the mats

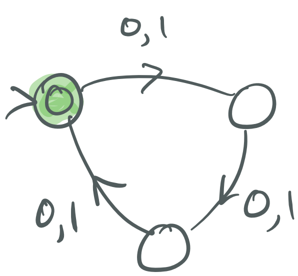
State of the door

	NEITHER	FRONT	REAR	BOTH
CLOSED	CLOSED	OPEN	CLOSED	CLOSED
OPEN	CLOSED	OPEN	OPEN	OPEN

More examples of FAS:



⇐ what kind of strings can drive the FA from the start state to end at a Final state



⇐ string 010 is accepted

string 0101 is not...

it is rejected.

Def$^{\underline{n}}$: A finite automaton is a 5-tuple $(Q, \Sigma, \delta, q_0, F)$ where:

Q is a finite set of **states**

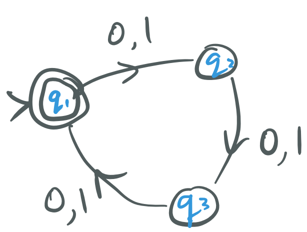
Σ is an **alphabet**

$\delta : Q \times \Sigma \to Q$ is a **transition function**

$q_0 \in Q$ is the **start state**

$F \subseteq Q$ is the set of **accept states**

Eg:



M

$M = (\{q_1, q_2, q_3\}, \{0,1\}, \delta, q_1, \{q_1\})$

where δ is given by

	0	1
q_1	q_2	q_2
q_2	q_3	q_3
q_3	q_1	q_1

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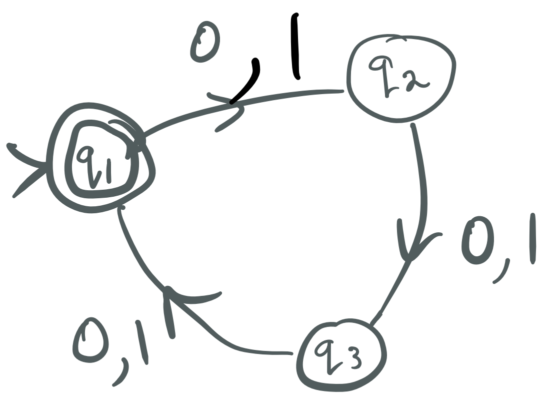
Eg:

$M = (\{q_1, q_2, q_3\}, \{0,1\},$

$q_1, \quad q_1)$



M

Def$^{\underline{n}}$: For a FA M whose alphabet is Σ

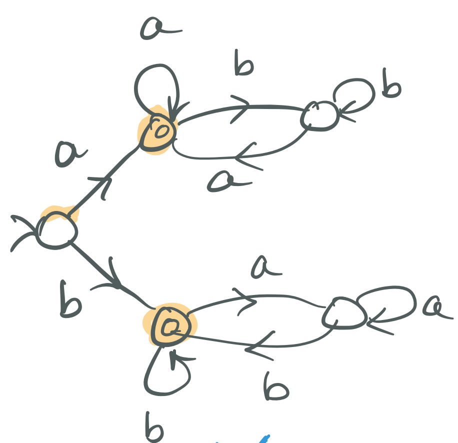
$L(M) = \{ w \mid w$ is a string over Σ and M accepts $w \}$.

We say $L(M)$ is the language recognized by (accepted by) M

Def$^{\underline{n}}$: FA M (over alphabet Σ)

accepts a string w if w "drives" M from start state to a final state.

E.g.



M′

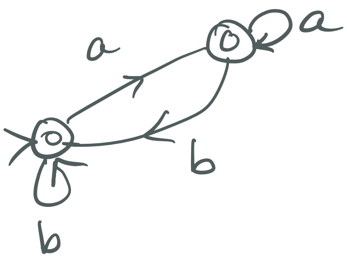
some strings accepted by M′:

a, aa, abbaba, ...

b, bab, bb, babbab, ...

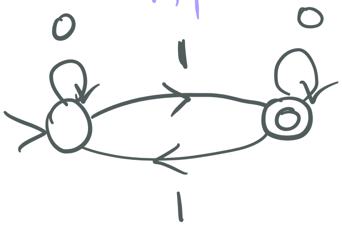
M' recognizes the language of strings over $\{a,b\}$ that start and end in same letter.

Does this FA also recognize the language of strings over $\{a,b\}$ that start and end in same letter?



Other examples:

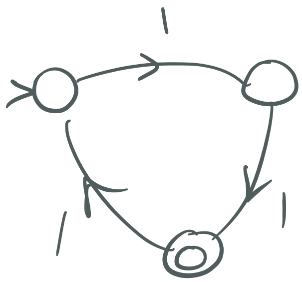
M_1



$\Sigma = \{0,1\}$

$L(M_1) =$

M_2

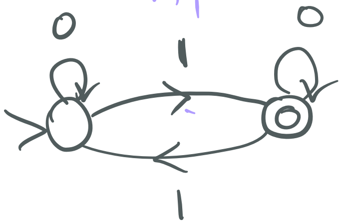


$\Sigma = \{1\}$

$L(M_2) =$

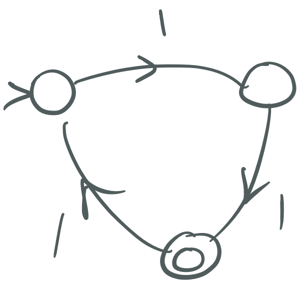
Other examples:

M_1



$\Sigma = \{0,1\}$ $L(M_1) = \{ \quad \}$

M_2

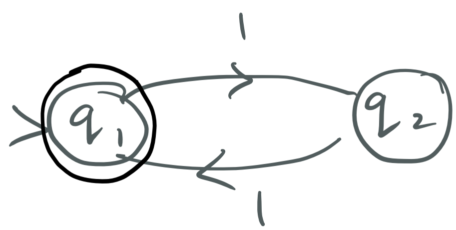


$L(M_2) = \{ \quad \}$

$\Sigma = \{1\}$

The "meaning" of a FA is the language it recognizes.

The "meaning" of a state can be regarded as the set of strings that can "drive" the FA from the start to that state.



M_5.

$L(M_5, q_1)$ = // language of strings that can drive
// M from start to q_1

is: 1. $\varepsilon \in L(q_1)$

2. if $s \in L(q_1)$, then so is $s11$

Nothing else is in $L(q_1)$.

$$L(M_5) = L(M_5, q_1).$$

In a way, a FA language is a recursively defined object:

$L(M_5)$ is $L q_1$ where

$L(M_5, q_1)$ is 1. ε

2. $w \cdot 1$ where $w \in L(M_5, q_2)$.

$L(M_5, q_2)$ is 1. $w \cdot 1$ where $w \in L(M_5, q_1)$

Claim: $L(M_5, q_1)$ is $\{ w : w$ is an even length string of 1's $\}$.

$L(M_5)$ is L_{q_2} where

$L(M_5, q_1)$ is 1. ε

2. $w \cdot 1$ where $w \in L(M_5, q_2)$.

$L(M_5, q_2)$ is 1. $w \cdot 1$ where $w \in L(M_5, q_1)$

Claim: $L(M_5, q_1)$ is $\{ w : w$ is an even length string of 1's $\}$.

Need a stronger claim.

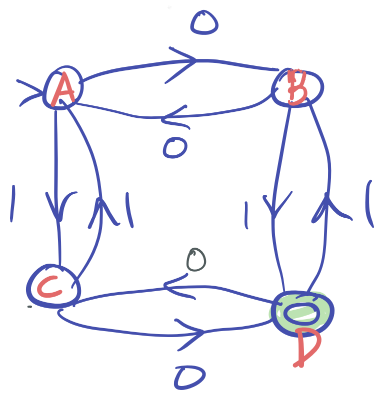
Claim: For any string w of 1's

$$(q_1, w) \vdash^* (q_1, \varepsilon) \text{ if } |w| \text{ is even}$$

and $(q_1, w) \vdash^* (q_2, \varepsilon)$ if $|w|$ is odd.

"yields, in 0 or more transition steps"

(More on this later)



What Language is recognized by this FA?

The Regular Operations on Languages

Defn: Let A and B be languages.

Define the following operations:

Union $A \cup B = \{x \mid x \in A \text{ or } x \in B\}$

Concat $A \circ B = \{xy \mid x \in A \text{ and } y \in B\}$

Star $A^* = \{x_1 x_2 \cdots x_k \mid k \geq 0 \text{ and } x_i \in A \ \forall i\}$

E.g. $A = \{ab, aa\}$ $B = \{bb\}$

$A \cup B = \{ab, aa, bb\} = \{bb, aa, ab\}$

$A \circ B = \{abbb, aabb\}$

$A^* = \{ \varepsilon, aa, ab, aaaa, aaab, abaa, abab, \ldots \}$

$B^* = \{ \varepsilon, bb, bbbb, \ldots \}$

note shortlex order

Defn A set A is closed under binary opn "$\blacksquare$" if $w_1 \blacksquare w_2 \in A$ whenever

$w_1 \in A$ and

$w_2 \in A$

Quiz: $\mathbb{N} = \{1, 2, 3, \ldots\}$

$\mathbb{W} = \{0, 1, 2, \ldots\}$

$\mathbb{Z} = \{\ldots -2, -1, 0, 1, 2, \ldots\}$

$\mathbb{N}$ closed under $+$?

$\mathbb{N}$ closed under $-$?

$\mathbb{W}$ closed under $\times$ (mult) ?

$\mathbb{W}$ closed under $\div$?

W closed under $-$ (minus)

W closed under $\boxed{\text{negation}}$?

Defn: A set is closed under unary op "Δ"

if $\Delta w \in A$ whenever $w \in A$.