2. Review Week 1 Jan 6 2026

From chapter 0 ...

Sets are like this : $\{a, b\}$

curly bracket

does not imply an order

If there is an (implied) "universe" $\mathcal{U}$, or ground set from which the set is drawn, then set complementation is defined

$$\overline{S} = \mathcal{U} \setminus S$$

"S complement"

all elements not in S

"set minus"

We also have $\cap$, $\cup$

Sequences and tuples - order is implied

- use ( )

"open paren" "close paren"

(Note : graph theory, undirected graphs, abuse this notation somewhat, using (u,v) for an undirected edge.)

Review: - Functions + relations
- Graphs.

STRINGS & LANGUAGES

Def$^{\underline{n}}$ An alphabet is a finite, non-empty set.
Its members are called symbols (or letters).

Σ "capital sigma"
Γ "capital gamma"
often used to denote alphabets

(α may denote an arbitrary symbol in the alphabet)

Def$^{\underline{n}}$ A string over an alphabet Σ is a finite sequence of symbols from Σ, duplicates allowed.

ε ("epsilon") is the symbol for the empty string.

$ab \cdot bb = abbb$
$S_1 \cdot S_2 = S_1 S_2$

Juxtaposing two strings is an operation called "concatenation", yielding a new string. Also use $\bullet$

Recursive Definitions.

Alternative definition of strings over $\{a,b\}$.

Defn: 1. ε is a string over $\{a,b\}$

2. if s is a string over $\{a,b\}$, so is $s \cdot a$

3. if s is a string over $\{a,b\}$, so is $s \cdot b$

4. Nothing else is a string over $\{a,b\}$

The above is a recursive definition of strings (over the given alphabet).

Once we have a recursive definition of an object, we can easily define functions of that object.

Def$^{\underline{n}}$ length of a string over $\{a, b\}$, denoted with abs-value $|s|$

1. $|\varepsilon| = 0$
2. $|s \cdot a| = |s| + 1$, where s is a string over $\{a,b\}$
3. $|s \cdot b| = |s| + 1$ " " "

Length is now defined for all strings over $\{a, b\}$, since (4) "Nothing else is a string over $\{a,b\}$"

Def^{n} "string over Σ".

1. ε is a string over Σ
2. if s is a string over Σ, and $\alpha \in \Sigma$ then $s \cdot \alpha$ is a string over Σ.
3. Nothing else is a string over Σ.

For you to do:

1. Give a definition of string length for strings over Σ.

2. We want $\#_a(s)$ defined on strings over $\{a, b\}$ to be the number of a's in s.
Make it so, using the recursive definition.

3. Do same for $\#_\alpha(s)$ for strings over Σ, $\alpha \in \Sigma$.

4. Prove using induction and the $\text{def}^{\underline{n}}$ of string length $|w|$ that
$|w_1 \cdot w_2| = |w_1| + |w_2| \quad \forall\, w_1, w_2$ over Σ