Computer Science 320 Midterm Test 1 NAME: Solutions

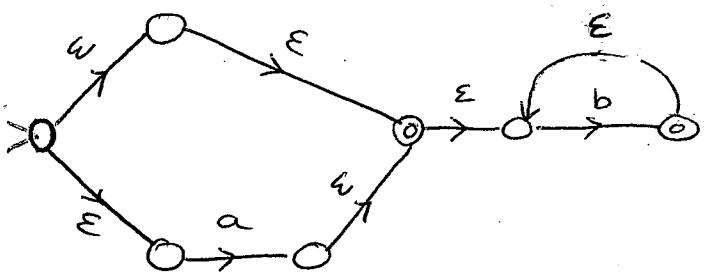
Jan 22 2015 Out of 57 marks

Recall that for a string w, $\#_\sigma(w)$ is the number of occurences of σ in w.

1. (1 mark) What is the closure of the set $\{6, 8\}$ under subtraction?

"Even integers" or $\{\ldots -4, -2, 0, 2, 4, \ldots\}$.

2. (4 marks) **Using the construction,** give the NFA that corresponds to the regular expression $(\epsilon + a)b^*$. **Do not** include a dead state. **Do** include all ϵ transitions that the construction dictates (even if they are not useful).



3. (10 marks) For each pair of regular expressions below, do they represent the same language? Answer True or False. If False, give a string that differentiates them (i.e., is in the language of one but not the other).

(a) aa^* and $(a + aa)^*$ — F ε

(b) $(ab^+ + b)^*$ and $(a^*b^*)^*$ — F a

(c) $a^* + b^*$ and $(a + b)^*$ — F ab

(d) $(a + b)^*ab(a + b)*$ and $(a^+b^+)^*$ — F bab, ε

(e) $((\epsilon + a)^*b)^*$ and $(a + b)^*b$ — F b, ε

4. (4 marks) Give a **regular expression** for the language of non-empty strings over $\{a, b\}$ that begin and end in the same letter and have a number of b's that is evenly divisible by 2.

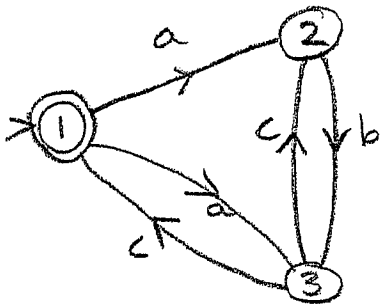
$a(a^*ba^*ba^*)^*a + a + b(a^*ba^*ba^*)^*b$

or

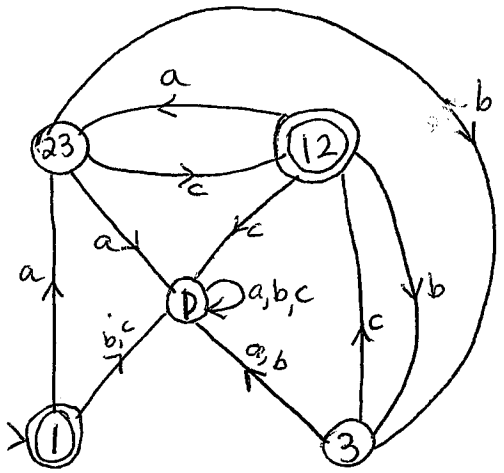
$a(a + ba^*b)^*a + b(a + ba^*b)^* + a$

(Reffel and Egorchatov solution)

5. (5 marks) Find, **using the construction,** a DFA that is equivalent to the following NFA. Show the dead state and all transitions to it.

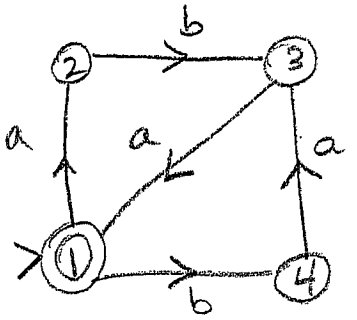


	a	b	c
1	2,3	∅	∅
2	∅	3	∅
3	∅	∅	1,2
1,2	2,3	3	∅
2,3	∅	3	1,2

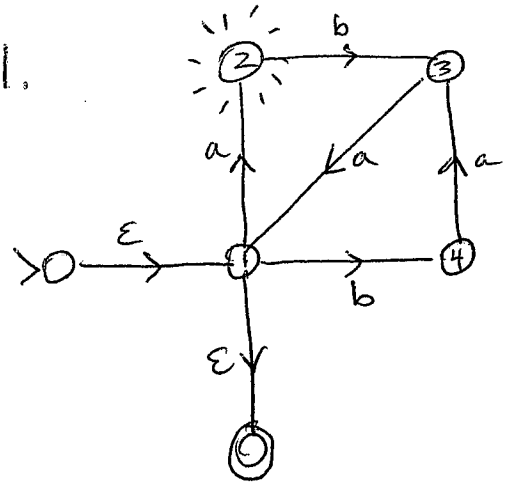


6. (5 marks) **Use the construction** to find the RE that describes the language accepted by the following FA.

missing transitions go to the dead state.

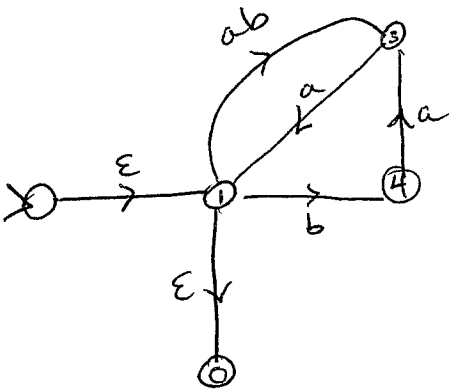


1.

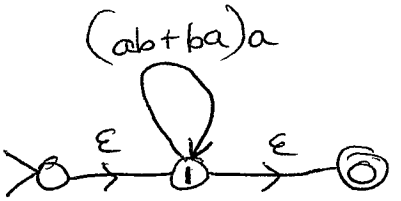


"Rip 2"

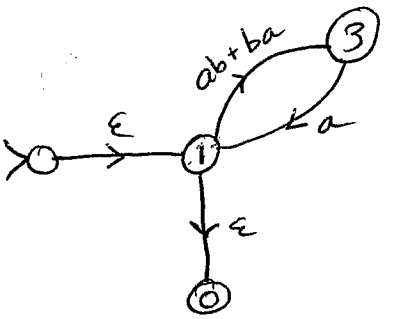
2.



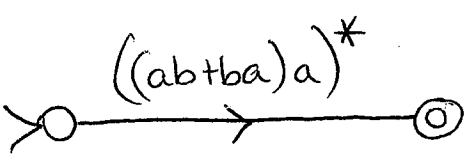
"rip 4"



"rip 3"



"rip 1"



re = $((ab+ba)a)^*$

7. Give a regular expression for the following languages:

   (a) (3 marks) $\{w \in \{a,b\}^* : w$ starts and ends with the same letter, and has at least one other occurance of that letter $\}$

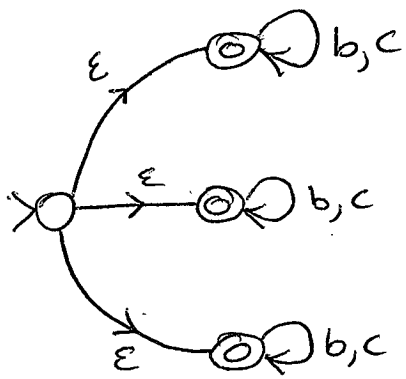
$$ab^*a(a+b)^*a \;+\; ba^*b(a+b)^*b$$

   (b) (3 marks) $\{w \in \{a,b\}^* : \#_a(w) \equiv 0 (\text{mod} 3)\}$

$$b^*(ab^*ab^*ab^*)^* \quad \text{or} \quad (b^*ab^*ab^*ab^*)^*b^*$$

8. (a) (4 marks) Draw a Non-deterministic finite automaton that accepts the following language:

$$\{w \in \{a,b,c\}^* : \#_a(w) = 0 \text{ or } \#_b(w) = 0 \text{ or } \#_c(w) = 0\}$$

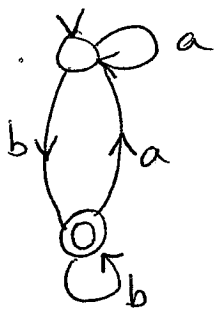


   (b) (2 marks) Give a regular expression for the same language.

$$(a+b)^* + (a+c)^* + (b+c)^*$$

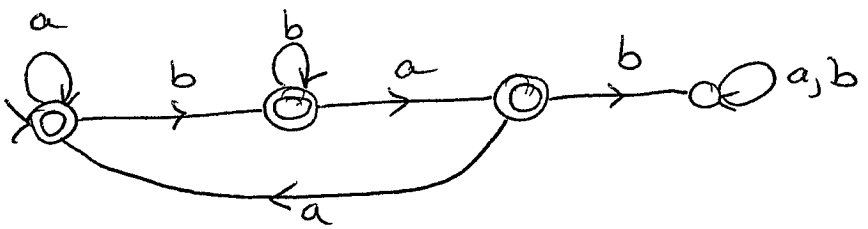
9. (a) (4 marks) Give a DFA for the language $L = \{w \in \{a,b\}^* : w$ ends in a $b\}$.

2 states are enough.



(4 marks)

(b) Give a DFA for the language $L = \{w \in \{a,b\}^* : w$ does not contain the substring $bab\}$. 4 states are enough.



(c) (4 marks) Let M_1 and M_2 be two DFA's, where

$$M_1 = \{Q_1, \Sigma, \delta_1, q_1, F_1\}$$

$$M_2 = \{Q_2, \Sigma, \delta_2, q_2, F_2\}$$

Show how to contruct the DFA $M' = \{Q', \Sigma, \delta', q', F'\}$ for the language $L(M_1) \cap L(M_2)$. That is, show the construction for the intersection of regular languages. Do so by giving the following:

$Q' := Q_1 \times Q_2$ ie. each state in M' is an ordered pair (p_1, p_2) where $p_1 \in Q_1$, $p_2 \in Q_2$

$q' := (q_1, q_2)$

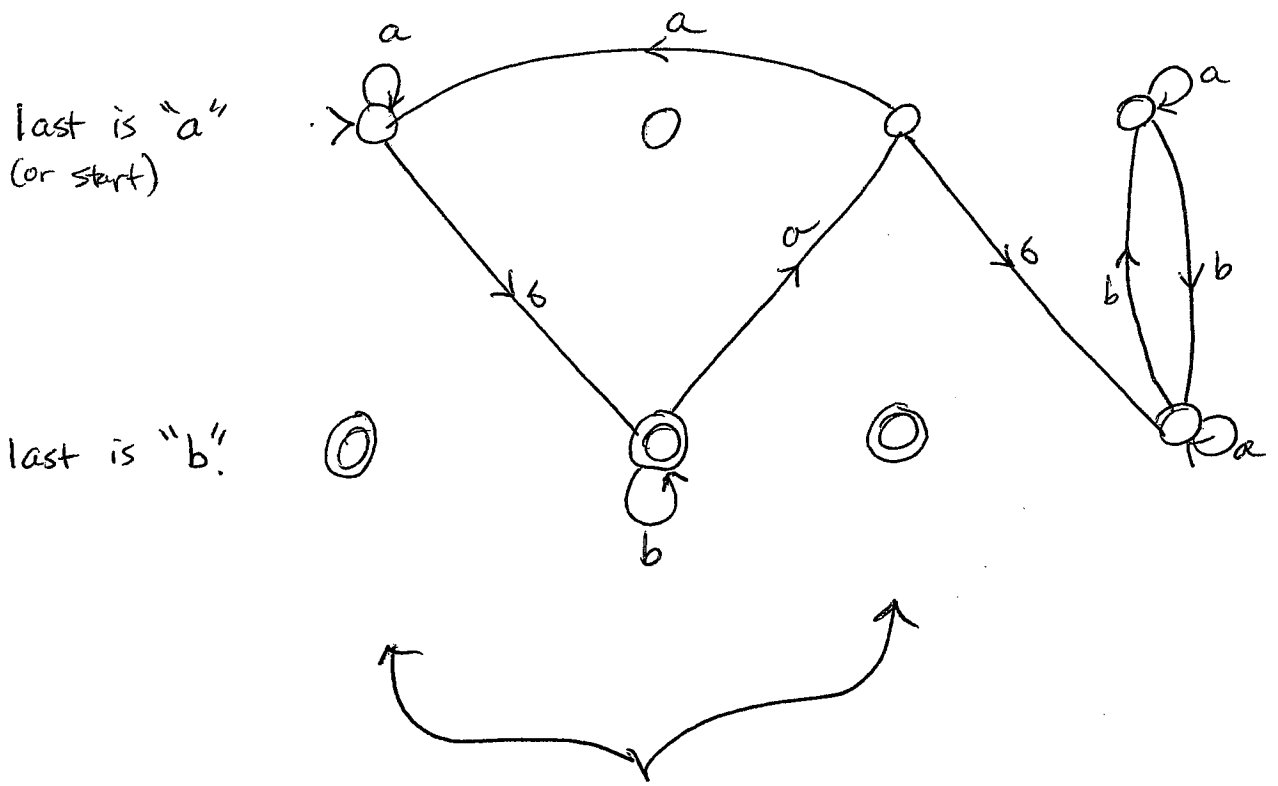
$F' := F_1 \times F_2$ ie both p_1 and p_2 must be accept states in their respective DFA's.

δ':

$$\delta'((p_1,p_2),\sigma) = (\delta_1(p_1,\sigma), \delta_2(p_2,\sigma))$$

You are encouraged to use the mathematical notation given in class, but it is also acceptable to describe the sets, ordered pairs, or functions above in precise English. (You may wish to do part (d) below first, as a model for your general solution.)

(d) (4 marks) Give a DFA for the language $L = \{w \in \{a,b\}^* : w$ ends in a b, and it does not contain the substring bab $\}$. That is, give a DFA for the intersection of the two languages from parts (a) and (b), above. For full marks, use the construction you outlined in part (c) if it is correct. For part marks, use the "grok and blurt" method to come up with a correct DFA.



these two states, and one above, are not reachable from the start state, so are not interesting and I have not shown the transitions into them, or out.