CSCI 260 2022 Assignment 1
Analysis of Algorithms, Master Theorem, Big-O
Submission Deadline: midnight, Saturday Oct 1, 2022.
Out of 38 marks

Handwritten is accepted in the original, not scanned. Electronically generated documents can be handed in electronically to gara.pruesse [at] viu.ca. 10% penalty for work submitted by Sunday at noon. 50% penalty for work submitted by Monday at noon. 100% penalty thereafter.

1. (1 marks) Explain briefly why the statement, "The running time of Algorithm A is at least $\mathbf{O}(n^2)$" is meaningless.

2. (4 marks) Prove the "Reciprocal Rule": $f(n) \in \mathbf{O}g(n) \Rightarrow \frac{1}{g(n)} \in \mathbf{O}(\frac{1}{f(n)})$.

3. (4 marks) Show that $3n^2 + 4n - \frac{\log n}{\log\log n} \in \mathbf{O}(n^2)$ using the Definition of Big-O.

4. (4 marks) Prove that $6n \lg n + 4n^3 - 8n \in \Theta(n^3)$. You can use the Rules or the definition of Big Oh.

5. (6 marks) Is $\frac{n}{\log n} \in \mathbf{O}(n)$? In $\Omega(n)$? In $\Theta(n)$? Answer Yes or No to each, and prove your answers using the Facts or the definitions of Big Oh, Ω, and Θ.

6. (4 marks) List the following functions in a column so that the function at the top grows the slowest, and the function at the bottom grows the fastest. If two functions grow at the same rate asymptotically, put them on the same row.

   n^n $\quad$ $n!$ $\quad$ $n + 10\sqrt{n}$ $\quad$ $(2.3)^n$ $\quad$ n^2 $\quad$ $2^{n \lg n}$ $\quad$ $10n \lg\lg n$
   2^n $\quad$ $n^2 \lg n$

7. Briefly describe what happens to the running time of an algorithm with run time $f(n)$ when the size of the input is increased by one, and when the size of the input doubles:

   (a) (2 marks) $f(n) = n$
   (b) (2 marks) $f(n) = n^2$
   (c) (2 marks) $f(n) = 2^n$

8. Solve the following Divide-and-Conquer recurrences using the Master Theorem.

   (a) (3 marks) $T(n) = 3T(n/4) + n$
   (b) (3 marks) $T(n) = 4T(n/2) + n^2$
   (c) (3 marks) $T(n) = 6T(n/3) + n \log n$