Computer Science 260 Practice for the Midterm3
Out of XX possible marks NAME:_____

1. (8 marks) Circle either **T** (True) or **F** (False), whichever is a more accurate assessment of the following statements:

   T **F** When open hashing, the number of slots must be at least as great as the number of keys to be stored.

   T **F** To rebalance and AVL tree after an insertion, we look for the highest node that is out of balance and do rotations at that node first.

   T **F** A good hash function guarantees that the keys will distribute uniformly among the slots.

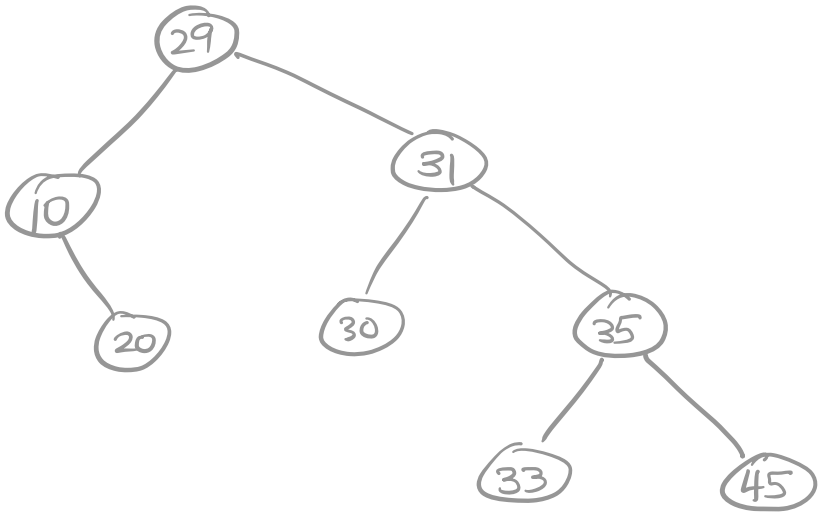
   T **F** All the Minimum Spanning Tree (MST) algorithms we studied are examples of greedy algorithms.

2. (6 marks) Use Dynamic Programming to find the longest common subsequence (not necessarily contiguous) in the following two strings, by filling the table below.

	λ	B	B	A	B	C	A	B	D
λ									
A									
B									
A									
C									
C									
A									
D									
B									

3. (5 marks) Perform the BST inserts of 12, 18, 13, 4, 5, 2, 11, 16, 15 into an originally empty BST. Do them in that order; show the tree that results. You only need to show the end result.

4. (5 marks) Execute an AVL-Tree insert of key 35 into the following AVL tree. Then label each node with its Bf value. Then AVL-insert 32.



5. (4 marks) Define 'clustering' and describe a strategy for avoiding it when using closed hashing.

6. Consider the hash function $h(c_1c_2\ldots c_n) = \Sigma_{i=1}^{n}\text{ord}(c_i)\text{mod}m$. Note that $\text{ord}(c_i)$ is the ordinal number of the letter c_i in the alphabet. The ordinal values are provided here for your convenience:

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	2
A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	

(a) (1 mark) What is does 'm' refer to in the above hash function?

(b) (6 marks) Insert the following keys into the hash table below, using open addressing (all elements are stored in the table itself) with double hashing to handle collisions. The secondary hash function is "string length + number of vowels" mod m. AN MB AR WX DE LN GI UF

(c) (2 marks) Why is it a bad idea if the value of m were 12? What kind of behaviour might it lead to in insertion and in deletion?

(d) (3 marks) Write the Find(key) algorithm for the above table. What happens if some elements have been deleted?

7. (6 marks) Give a simple, efficient, recursive, Dynamic Programming algorithm to perform exponentiation. Hint: $a^n = a * a^{\frac{n}{2}} * a^{\frac{n}{2}}$, if n is odd, and $a^n = a^{\frac{n}{2}} * a^{\frac{n}{2}}$, if n is even. Integer division, rounding down, is used in the exponents.

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global variable int aToTheN[n+1] is declared and initialized to zeroes.

Algorithm Raise( a, n )
input: a is a floating point number, n is an non-negative integer
output: a^n
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8. Give a set of coin denominations for which the greedy algorithm does <u>not</u> provide the optimal (minimum) number of coins. Prove your claim by giving an amount C, and the change made by the greedy algorithm, and a way to make the change that uses fewer coins.

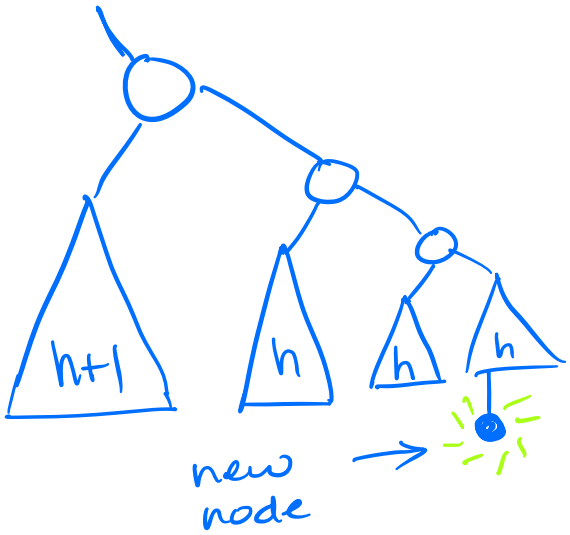
9. Describe how you would efficiently find the record with minimum key value for each of the following Data Structures, and give the asymptotic running time for the algorithm. Your description can be a single line (or more, if necessary) in English, or pseudocode. CLARITY is important! Assume that the insert algorithms are the standard ones, and cannot be changed.

   (a) (4 marks) AVL tree of n nodes.

(b) (4 marks) MinHeap of n nodes.

(c) (4 marks) Hash table, table size m, number of keys in it is n where $n \leq m$, and the Universe of key values is 2^{10}, which is much bigger than m; and Open Addressing (all elements stored in table) is used.

10. (5 marks) Suppose an element is inserted into an AVL tree and its position, before rebalancing, is as shown below, with relative heights as shown. Perform the necessary rebalancing and give the AVL tree, with nodes and subtrees labelled, that results.



11. (5 marks) Put the following values into an AVL tree, in the order given.

    15, 4, 2, 9, 7, 6, 18, 19.

12. Hash Tables

(a) (2 marks) What best describes the ADT that hashing is designed to implement: Dictionary, Priority Queue, Graph, Heap, Skip List, Sparse Table, Ink Blot, Binary Counter.

(b) (4 marks) Describe in general terms the strategy for handling collisions utilized by Open Hashing (Separate Chaining).

(c) (4 marks) Describe Open Hashing (Separate chaining). Under what circumstances is it preferred to Open Addressing (where all elements are stored in the table)?

(d) (4 marks) Write the pseudocode for ChainedHashDelete($T, x,$), where T is a hash table, x is the key value of the item to be deleted, and collisions are handled by chaining.

(e) (5 marks) Under the assumption of Simple Uniform Hashing, if x and y are two keys selected from a set of keys K, where K is a subset of the Universe of keys U, and the hash table is of size m, determine the following, in terms of m, $|K|$ and $|U|$.

(f) the probability that x and y hash to the same slot

(g) In the following situation, if Open Addressing with and quadratic probing is used, where $h(k,i) = (h'(k) - 2i + i^2) \text{mod} 11$, show the result of inserting A, B, C, and D, in that order, if $h'(A) = 4, h'(B) = 6, h'(C) = 10, h'(D) = 3$.