"Advanced" Master Theorem.

$$T(n) = a\,T\left(\tfrac{n}{b}\right) + \cdot\, n^{q} \lg^{p} n$$

$$T(n) = aT\left(\tfrac{n}{b}\right) + f(n).$$

$$T(n) = 9T(n/3) + \underbrace{n}$$

$\log_3 9 = \underline{\underline{2}}$

$f(n) = n$

$n \in$? $O(n^{2-\varepsilon})$

$\Theta(n^2)$

$\Omega(n^{2+\varepsilon})$

$\underline{\underline{n^2}}$

$\Rightarrow$ case 1 $\Rightarrow \Theta(n^2)$

Master Theorem: $T(n) = aT(n/b) + f(n)$

$\Rightarrow$ case 1: $f(n) \in O(n^{\log_b a - \varepsilon}) \Rightarrow T(n) \in \Theta(n^{\log_b a})$

case 2: $f(n) \in \Theta(n^{\log_b a}) \Rightarrow T(n) \in \Theta(n^{\log_b a} \lg n)$

$\rightarrow$ case 3: $f(n) \in \Omega(n^{\log_b a})$ AND (regularity)

$\Rightarrow T(n) \in \Theta(f(n))$

$T(n) = T\left(\frac{2n}{3}\right) + \underline{1}$ $\quad f(n) = 1 \quad n^{\log_b a}$

$b = 3/2 \quad a = 1.$

$O(n^{0-\varepsilon})$

$$1 \overset{?}{\in} \Theta(n^0) \quad n^{\log_{3/2} 1} = n^0.$$

$$\Omega(n^{0+\varepsilon})$$

case 2 so $T(n) \in \Theta(\lg n)$

Shuffle(A[1 ... n])

Shuffle(A[1 .. n/2])

Shuffle(A[n/4 .. 3n/4])

Shuffle(A[n/2 ... n])

Do $\Theta(n^2)$ other work.

$$T(n) = 3T\left(\frac{n}{2}\right) + n^2 \qquad f(n) = n^2.$$

$$n^{\log_b a} = n^{\log_2 3} \approx n^{1.58}$$

$$n^2 \in \overset{O}{\Theta}(n^{1.58})$$

$$\Omega(n^{1.58+\varepsilon})$$

Check regularity condition. $\exists c \quad 0 < c < 1$

s.t. $a f\left(\frac{n}{b}\right) < c \cdot f(n). \quad \forall n \geq a$

$$3\left(\frac{n}{2}\right)^2 = \frac{3}{4} n^2 < c \cdot n^2 \; ?$$

Yes $c = .8$ works.

$\therefore$ by Case 3 of MT, $T(n) \in \Theta(n^2)$.