

"Advanced" Master Theorem.

$$T(n) = a T\left(\frac{n}{b}\right) + n^q \lg^p n$$

$$T(n) = a T\left(\frac{n}{b}\right) + f(n).$$

$$T(n) = 9 T\left(\frac{n}{3}\right) + n$$

$$f(n) = n$$

$$\log_3 9 = 2$$

$$\begin{aligned} &? O(n^{2-\epsilon}) \\ n &\in \Theta(n^2) \\ &\Omega(n^{2+\epsilon}) \end{aligned}$$

$$\underline{\underline{n^2}}$$

$$\Rightarrow \text{case 1} \Rightarrow \Theta(n^2)$$

Master Theorem: $T(n) = a T\left(\frac{n}{b}\right) + f(n)$

$$\Rightarrow \text{case 1: } f(n) \in O(n^{\log_b a - \epsilon}) \Rightarrow T(n) \in \Theta(n^{\log_b a})$$

$$\text{case 2: } f(n) \in \Theta(n^{\log_b a}) \Rightarrow T(n) \in \Theta(n^{\log_b a} \lg n)$$

$$\begin{aligned} \rightarrow \text{case 3: } f(n) \in \Omega(n^{\log_b a}) \text{ AND (regularity)} \\ \Rightarrow T(n) \in \Theta(f(n)) \end{aligned}$$

$$T(n) = T\left(\frac{2n}{3}\right) + 1$$

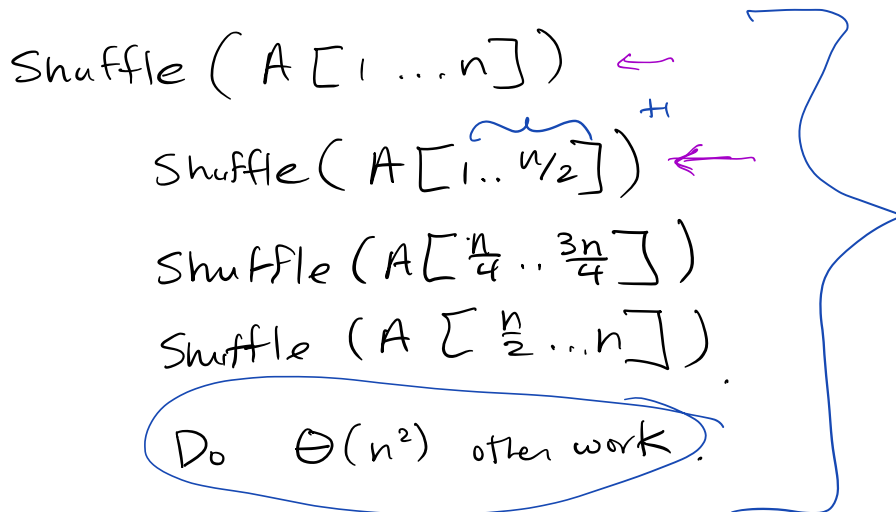
$$f(n) = 1 \quad n^{\log_b a}$$

$$b = \frac{3}{2} \quad a = 1.$$

$$n(n^{0-\epsilon})$$

$$1 \in \Theta(n^0) \leftarrow \Omega(n^{0+\epsilon}) \quad n^{\log_{3/2} 1} = n^0.$$

case 2 so $T(n) \in \Theta(\lg n)$



$$T(n) = 3T\left(\frac{n}{2}\right) + n^2$$

$$f(n) = n^2.$$

$$n^2 \in \Theta(n^{1.58})$$

$$n^{\log_b a} = n^{\log_2 3} \approx n^{1.58}$$

$$\Omega(n^{1.58+\epsilon}) \leftarrow$$

Check regularity condition. $\exists c \ 0 < c < 1$

$$\text{s.t. } a f\left(\frac{n}{b}\right) < c \cdot f(n), \quad \forall n \geq 0$$

$$3 \left(\frac{n}{2}\right)^2 = \left(\frac{3}{4}\right) n^2 < c \cdot n^2 ?$$

Yes $c = .8$ works.

∴ by case 3 of M.T, $T(n) \in \Theta(n^2)$.