

Note about Master Theorem...

Even our advanced Master Theorem does not have all the answers for all Divide & Conquer recurrence relations.

Consider $T(n) = 2T(n/2) + \underbrace{n \lg n}_{f(n)}$ $\log_2 2 = 1$

$$\begin{aligned} n \lg n & \stackrel{?}{\in} O(n^{1-\epsilon}) \\ & \in \Theta(n) \\ & \in \Omega(n^{1+\epsilon}) \end{aligned}$$

$n \lg n$ is not in any of these categories

We can see that more clearly if we

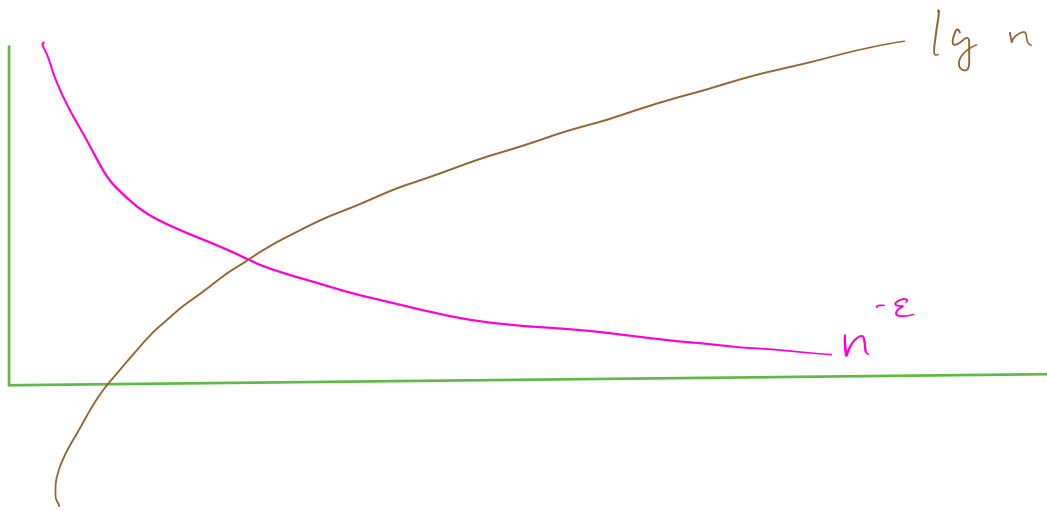
divide $n \lg n$ and what's in the O, Θ, Ω

by n (yes, we can do that)

Then the question becomes: for positive ϵ

$$\begin{aligned} \lg n & \stackrel{?}{\in} O(n^{-\epsilon}) \\ & \in \Theta(1) \\ & \in \Omega(n^{\epsilon}) \end{aligned} \quad \left. \begin{array}{l} \text{we know } \lg n \\ \text{is not in these} \\ \text{classes.} \end{array} \right\}$$

$n^{-\epsilon}$ is actually a decreasing function,
whereas $\lg n$ is an increasing function,
so clearly $\lg n \notin O(n^{-\epsilon})$ for any positive ϵ .



◦ The master Theorem is SILENT on
this recurrence.



That's the answer you give on a test
or assignment.