II Hashtables: Open Addressing

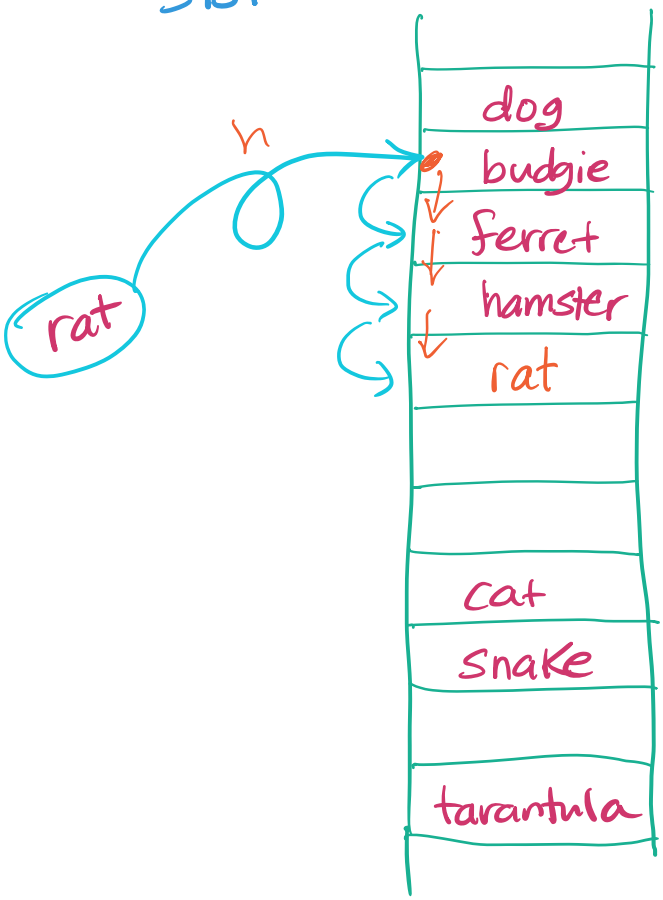
- all elements (or pointers to them) are stored in the table itself

$\Rightarrow \alpha \leq 1$

$\alpha = \frac{n}{m}$ (n: # elements, m: # slots)

Simple method of handling collisions:

- add 1 to slot # until find an empty slot



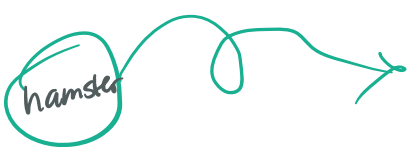
To find a key K:

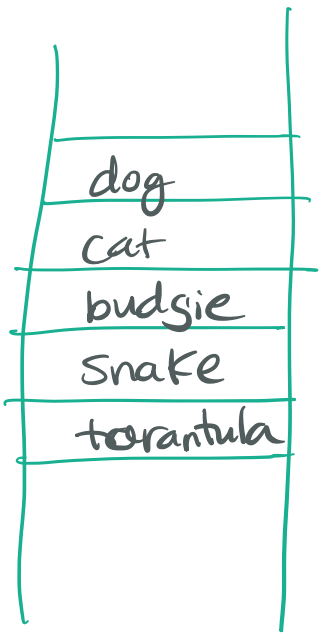
- successively probe slots starting at h(K) until either find the key or encounter an empty slot

To Insert(K), do the same but when encounter an empty slot, store element there.

Problem: "Clustering"

- a few collisions will fill adjacent slots
  - as cluster grows, the cluster is more likely to be location of a collision, and becomes more likely to grow....





The bigger the cluster gets, the more likely you are to add to it.

In our simple scheme, once you hash to a cluster, the operations take time Θ(size of cluster)

We can do better by hashing to determine the probe sequence.

probe sequence for key k
- slots searched when looking for key k.

Our simple strategy above yields a probe sequence of

$$\langle h(k), h(k)+1 \bmod m, h(k)+2 \bmod m, \ldots \rangle$$

Instead, lets use a different hash function depending on the probe:

$$\langle h(k,0), h(k,1), h(k,2), \ldots h(k,m-1) \rangle$$

We require that the probe sequence for any key be a permutation of

0 1 2 3 ... m-1

That way, we guarantee that

if ∃ an empty slot ⇒ an Insert will eventually find it and insert new key.

```
HashInsert (e, k)   // e an element with key k

for (int i = 0; i < m; i++)
    slot = h(k, i)
    if T[slot] = NULL
        T[slot] = (e, k)
        return slot
error "hash table overflow"
```

Linear probing

auxiliary hash function.

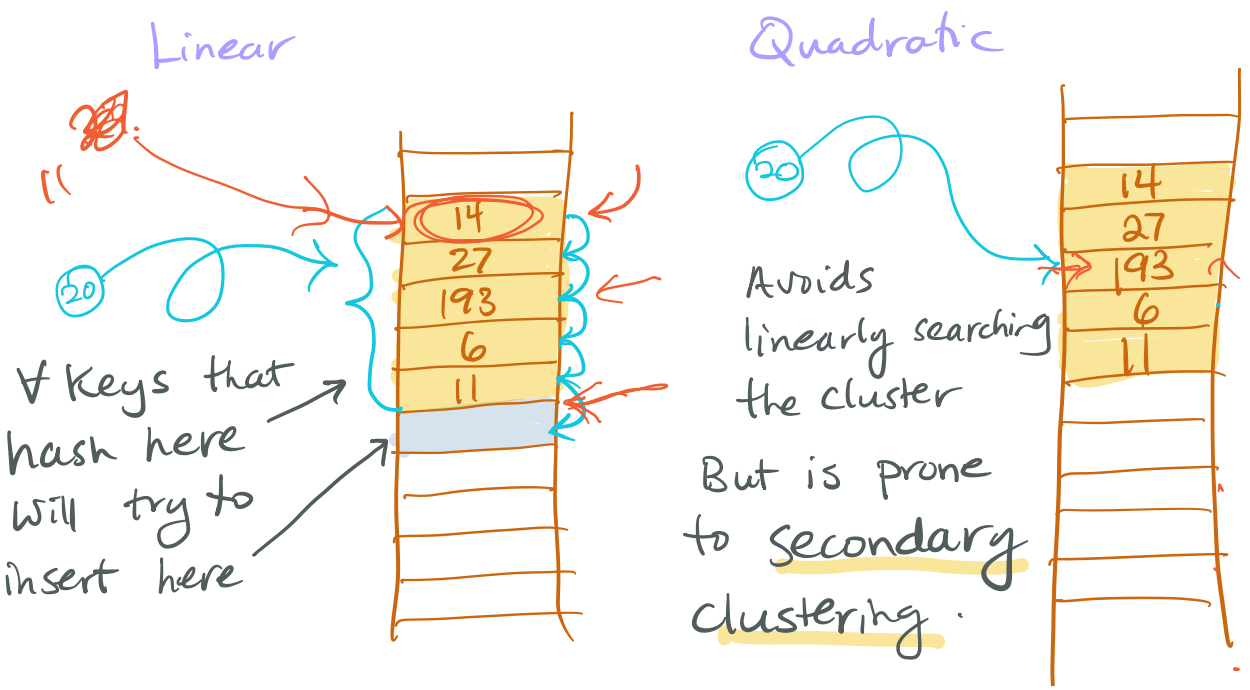
$$h(k,i) = \left(h'(k) + i\right) \bmod m$$

ie. this is our simple strategy and it has the problem of "primary" clustering: long runs of contiguous occupied slots

Quadratic probing

$$h(k,i) = \left(h'(k) + c_1 i + c_2 i^2\right) \bmod m$$

Better:



Double Hashing

$$h(K,i) = \left(h_1(K) + i\, h_2(K) \right) \bmod m$$

h_1, h_2 are auxiliary hash functions.

Two keys K_1 and K_2 may have a collision: $h_1(K_1) = h_1(K_2)$

But what are the chances they also collide in their second probe? $h_2(K_1) \stackrel{?}{=} h_2(K_2)$

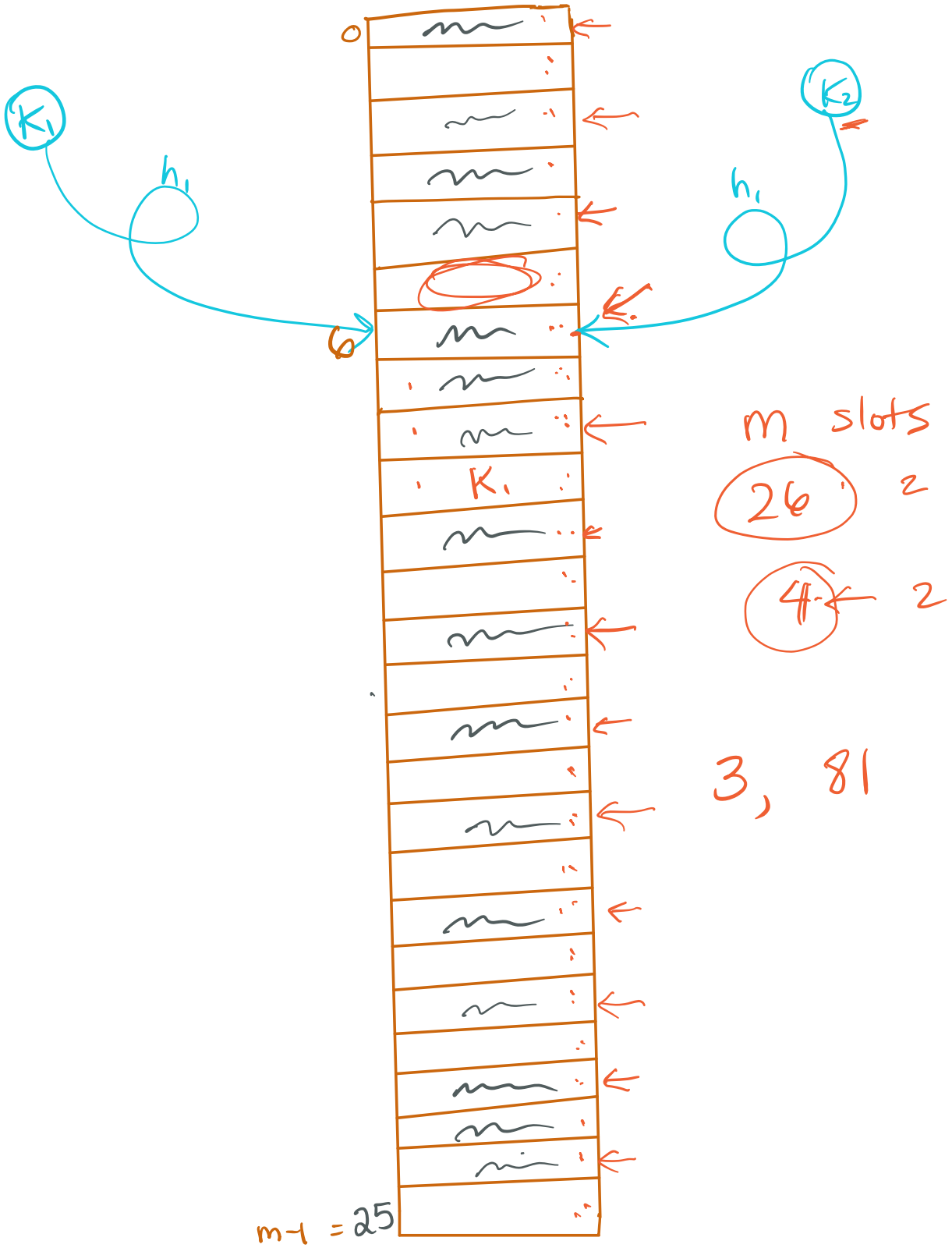
$\Rightarrow \frac{1}{m}$ likelihood

... low if you select them well (so their outputs "look independent")

$h_2(K)$ = "offset" for successive probing.

eg if $h_1(K_1) = h_1(K_2) = 6$

and $h_2(K_1) = 3$ and $h_2(K_2) = 4$

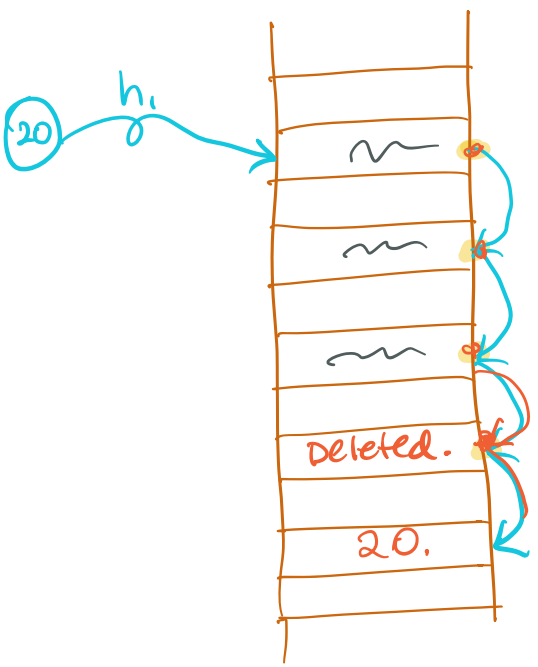

0
K1
h1
6
K2
h1
K1
m slots
26
2
4
2
3, 81
m-1 = 25

The value of $h_2(k)$ must be relatively prime with m in order that the probe sequence is a permutation of $\{0, \ldots, m-1\}$

- ie so that $\exists$ empty slot $\Rightarrow$ probing will find it.

Easy solution: Use prime m

How to Delete when Double Hashing?



Just set to NULL?

Insert(20)

Delete(30)

Find(20)

- write Sentinal that means DELETED into slot after deletion.
- This can be overwritten

during Insert.

How good is it?

Theorem 12.5 (CLRS)

Assuming uniform hashing, and open-address hash table with load factor $\alpha = \frac{n}{m} < 1$,

$\Rightarrow$ # of probes expected in unsuccessful search is $\leq \frac{1}{1-\alpha}$

Insert takes same time as unsuccessful search

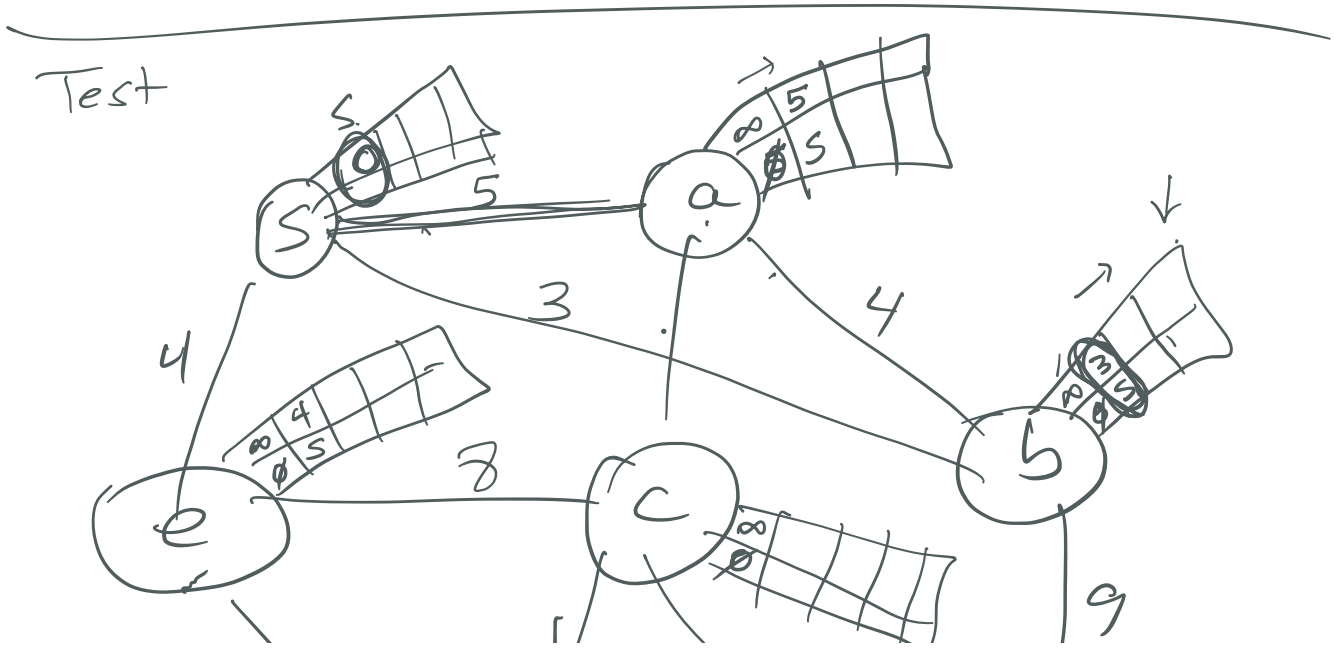
Theorem 12.7 (CLRS)

Assuming uniform hashing, and open address hash table with load factor $\alpha = \frac{n}{m} < 1$,

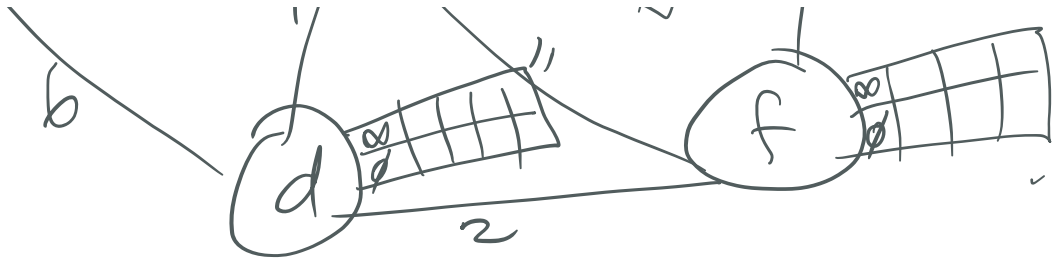
$\Rightarrow$ # of probes expected in successful search is $\frac{1}{\alpha} \ln \frac{1}{1-\alpha} + \frac{1}{\alpha}$

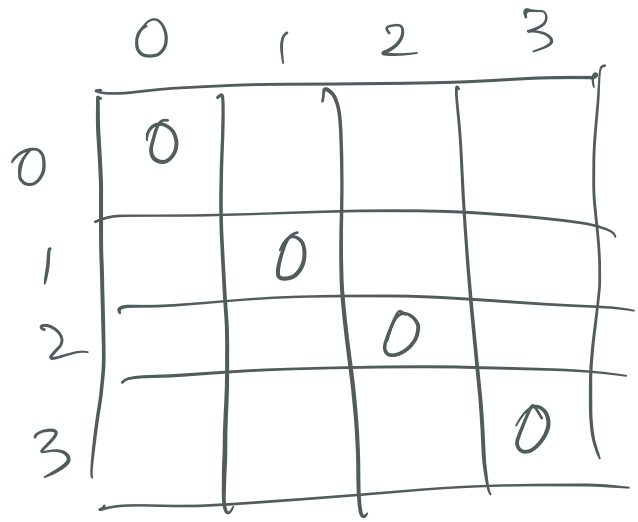
∘∘ if the hashtable is half full
then expect ≤ 3.39 probes to success!

90%: $\frac{1}{.9} * \underbrace{\ln(10)}_{2.3} + \frac{1}{.9} \approx 3.67$



Table





$O(n^n)$

Θ

$\Theta(n+m)$ is size of Adj ~~Matrix~~ Lists.

$\Theta(n^2)$ is " " Adj Matrix

$|E|$

$f(n,m)$

e

$m + n \log n$

edges # vertices.