Tutorial

$SAT = \{ \langle \phi \rangle \mid \phi$ is a Boolean expression in CNF that has a satisfying assignment $\}$

$3SAT = \{ \langle \phi \rangle \mid \phi$ is a Boolean expression in 3-CNF that has a satisfying assignment $\}$

CNF: "Conjunctive Normal Form" $= (\cdot \vee \cdot \vee \cdot) \wedge (\cdot \vee \cdot) \wedge \cdots (\cdot \vee \cdot \vee \cdot \vee \cdot)$

$\wedge$ and

$$\phi = (\bar{x} \vee y) \wedge (x \vee \bar{y} \vee \bar{z}) \wedge (\bar{x} \vee \bar{z} \vee \bar{y}) \wedge (x \vee y \vee \bar{z}) \wedge (x \vee z) \wedge (\bar{x} \vee \bar{z})$$

$\boxed{\text{Q1: Is there an assignment of } T, F \text{ to } x, y, z \text{ that makes } \phi \text{ true?}}$

3CNF = the clauses have exactly 3 literals each.

$$\phi' = (\bar{x} \vee \bar{x} \vee y) \wedge (x \vee \bar{y} \vee \bar{z}) \wedge (\bar{x} \vee \bar{z} \vee \bar{y}) \wedge (x \vee y \vee \bar{z}) \wedge (x \vee x \vee z)$$

Converting a Boolean formula from CNF to 3CNF form:

$$(x \vee \bar{y}) \Rightarrow (x \vee \bar{y} \vee \bar{y})$$

$$(x \vee \bar{y} \vee z \vee \bar{w}) \Rightarrow (x \vee \bar{y} \vee A) \wedge (\bar{A} \vee z \vee \bar{w})$$

$$(x \vee y \vee z \vee w \vee u) \Rightarrow (x \vee y \vee \quad) \wedge (\quad \vee z \vee \quad) \wedge (\quad \vee \quad \vee \quad)$$

Q2: Convert $\phi_1 = (\bar{x} \vee y \vee \bar{z} \vee u \vee \bar{v} \vee w)$ to 3CNF in a manner that preserves its satisfiability

$$\phi = (\bar{x} \vee y) \wedge (x \vee \bar{y} \vee \bar{z}) \wedge (\bar{x} \vee \bar{z} \vee \bar{y}) \wedge (x \vee y \vee \bar{z}) \wedge (x \vee z) \boxed{\wedge (\bar{x} \vee z)}$$

Q1: Is there an assignment of T, F to x, y, z that makes ϕ true?

Q3: Is the CNF ⇒ 3CNF conversion poly-time (on the size of the input ie number of literals occurring in ϕ)?

Q4: Show that DHamCycle $\leq_p$ HamCycle.

Hint: $\forall$ vertex v in directed graph G, replace w/ v_{in} — v_{mid} — v_{out} in undirected graph G'.

How will you connect these vertex-gadgets so that G has a DHamCycle $\Leftrightarrow$ G' has a HamCycle?