Assignment 4

Due April 9, 2026

1. (8 marks) Give a 3-SAT boolean expression that is satisfiable iff the following SAT expression is satisfiable. Use the construction studied in class.

$$\phi = (x \vee \bar{y} \vee z \vee \bar{w} \vee u \vee \bar{t}) \wedge (\bar{x} \vee t)$$

2. Recall that the language IndepSet is defined as follows:

$$\text{IndepSet} = \{ \langle G, k \rangle \mid G \text{ is an undirected graph that has an independent set of size } k \}$$

Recall that an indep. set is a set of vertices, no two of which are adjacent.

a) (4 marks) Prove that IndepSet is $\in$ NP.

b) (4 marks) Prove that IndepSet is $\in$ NP-c (i.e., is NP-complete).

3. (4 marks) Is the following formula satisfiable?

$$(x \vee y) \wedge (x \vee \bar{y}) \wedge (\bar{x} \vee y) \wedge (\bar{x} \vee \bar{y})$$

If so, give the satisfying assignment.

If not, argue why.

4. (4 marks) Show that NP is closed under concatenation.

5. (4 marks)

$$\text{SetPartition} = \{ \langle S \rangle \mid S \text{ is a set of integers such that } \exists\, A \subseteq S \;\; B \subseteq S \text{ where } S = A \cup B, \text{ and } A \cap B = \phi \text{ and } \sum_{a \in A} a = \sum_{b \in B} b \}$$

(i.e. S can be partitioned into two parts, each having the same element sum. Eg $\{1, 3, 5, 8, 9\}$ is in SetPartition but $\{1, 2, 3, 9\}$ is not.

Is SetPartition in NP? If so, prove your answer. If not, give your rationale.

6. NoSetPartition $= \{ \langle S \rangle \mid S$ is a set of integers such that $\nexists A \subseteq S \quad B \subseteq S$ where $S = A \cup B$, and $A \cap B = \emptyset$ and $\sum_{a \in A} a = \sum_{b \in B} b \}$

(i.e., $\forall$ partitions of S into two sets A and B, the sum of the elements of A will not equal the sum of the elements of B.

Eg, $\{1, 2, 3, 9\} \in$ NoSetPartition

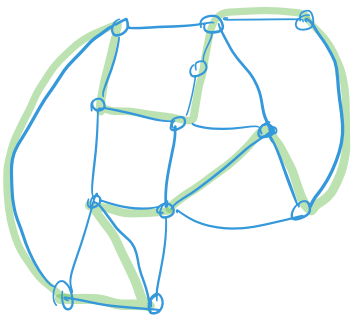
$\{1, 3, 5, 8, 9\} \notin$ NoSetPartition

(2 marks) Is NoSetPartition $\in$ NP ?

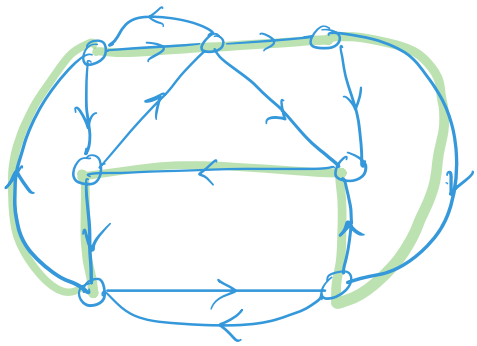
7. Show that HamCycle reduces in poly time to DHamCycle.

HamCycle = $\{ \langle G \rangle \mid G$ is an undirected graph that has a Hamilton cycle $\}$

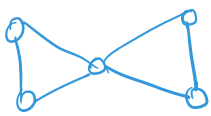
DHamCycle = $\{ \langle G \rangle \mid G$ is a directed graph that has a Hamilton cycle $\}$



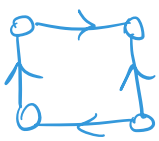
A graph $\in$ HamCycle



a graph $\in$ DHamCycle.



A graph $\notin$ HamCycle



a graph $\notin$ DHamCycle.